6

MATHEMATICS 24

Module 6
DESIGN and CONSTRUCTION

Learning Technologies Branch

Alberta LEARNING

MATHEMATICS 24

Module 6
DESIGN and CONSTRUCTION

Mathematics 24
Module 6: Design and Construction
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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Welcome to

MATHEMATICS 24

Module Six

Mathematics 24 contains six modules. You should work through the modules in order (from 1 to 6) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Module 1 contains general information about the course components, required resources, visual cues, assessment and feedback, and strategies for completing your work. If you do not have access to Module 1, contact your teacher to obtain this important information.

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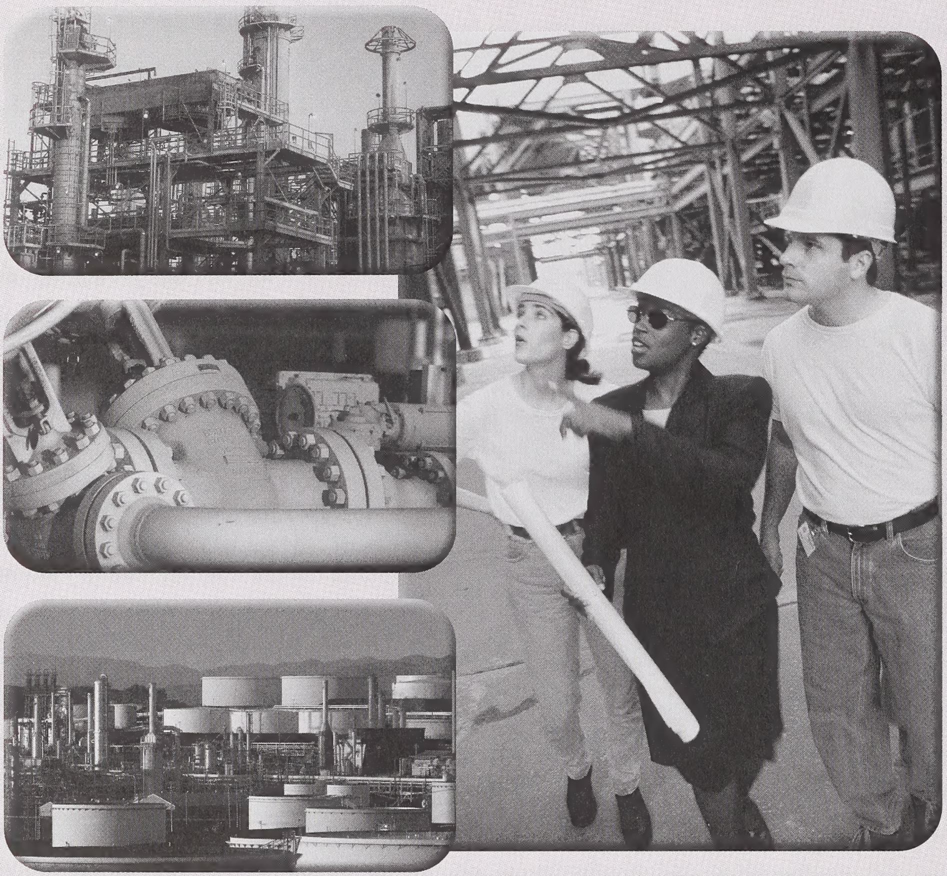
MODULE OVERVIEW



Industrial complexes and petroleum refineries seem to be an endless array of pipes, towers, and containers. There are valves here and tanks there—it's all far too complicated for anyone to comprehend each and every detail. Yet these buildings and structures work just the way their engineers expected them to.

In the modern world, projects are designed by teams that use computers to help prepare the plans. Every detail is looked after, right down to the size of the fasteners used, their length, and their strength. How can this massive amount of information be represented and then used?

In this module you will explore ways of representing the three-dimensional world in simpler ways. You will also find ways to fit small objects into larger containers, and you will use the properties of circles and polygons to make interesting and useful designs.

You must use the PDF file called "Module 6: Section 2: Lesson 1 and Related Diagrams" on the multimedia CD in order to complete Section 2: Lesson 1 and the related Assignment Booklet questions. You may either work from your computer or print the PDF file on a colour printer.

Module 6
DESIGN and CONSTRUCTION

Section 1
PRISMS and NETS

Section 2
DRAWINGS and DESIGN

Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The suggested mark distribution is as follows. Be sure to check with your teacher if this mark allocation is valid for you. Some teachers like to include other reviews and assignments.

Assignment Booklet 6A	
Section 1 Assignment	43 marks
Assignment Booklet 6B	
Section 2 Assignment	46 marks
Final Module Assignment	11 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 6A to your teacher before you begin Section 2. You will submit Assignment Booklet 6B to your teacher at the end of this module.

SECTION 1

Prisms and Nets

One of the basic human needs is shelter. This need has been met many different ways throughout history, but one of the most predominant ways is with tents. The shapes and supports of tents have varied throughout history, depending on their function. Many different societies used tents with very similar shapes for shelter. Invading armies housed troops in tents over 2000 years ago. Antarctic explorers use tents as shelter even today.

Modern tents have more shapes than ones used 50 or 60 years ago. Older tents seem to have one of two main shapes—either pyramids or prisms. These shapes may have been used because they were easy to support and they were straightforward to build. They had a central frame and a covering made of triangles or rectangles.

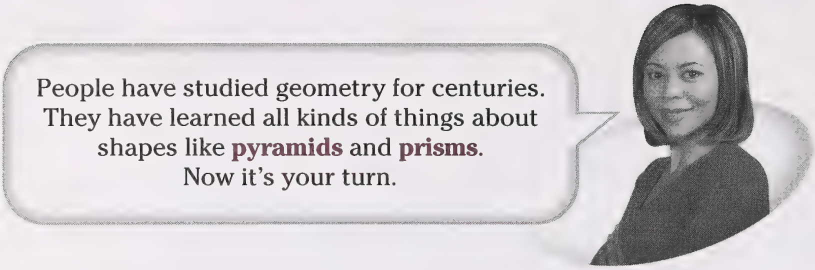
In this section you will study pyramids and prisms. You will see how their shapes can be built from triangles and rectangles. These basic building pieces can be made into nets that cover the outside of the shape. You will also look at nets for more complex shapes.

LESSON 1

Prisms and Pyramids

In this lesson you will learn how to classify objects as prisms and pyramids.

Ginnette was looking at the pictures her grandparents brought back from their trip around the world. Her grandfather had organized the pictures according to themes. These photos show some of the basic structural shapes used by people. The ancient pyramids of Egypt and Mexico and the modern pyramids used for office towers and other purposes don't appear to be too different. The building materials may have changed, but the shape and grandeur remain. Why people started using these shapes will never be known for sure. It is unlikely that it is because they saw the same shapes in nature.



Turn to page 311 in your textbook and read the "get thinking" box at the top of the page. Once you have started to think about the shapes of things in the world around you, you will be ready for this lesson.

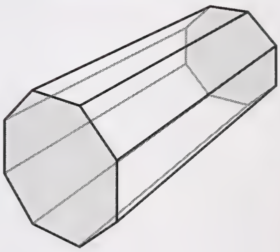
Look at the images on page 311. They are divided into two groups—the prisms and the pyramids. How are these two groups different? How are the items in each group similar?

1. Turn to page 312 in your textbook and do questions 1 to 3 of "Investigation: Prisms or Pyramids?" (If you don't have a partner, try to get a parent or friend to help you. Otherwise, answer the questions on your own.

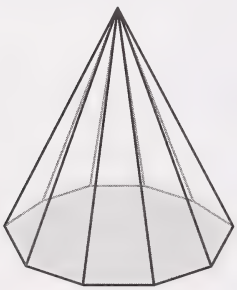
Check your answers on page 33 in the Appendix.

Example

Is the shape to the right a pyramid or a prism?



Notice that two ends of the shape are parallel and congruent. Each end has eight sides. The other sides of the shape are rectangles. These facts tell you that the shape is a prism.

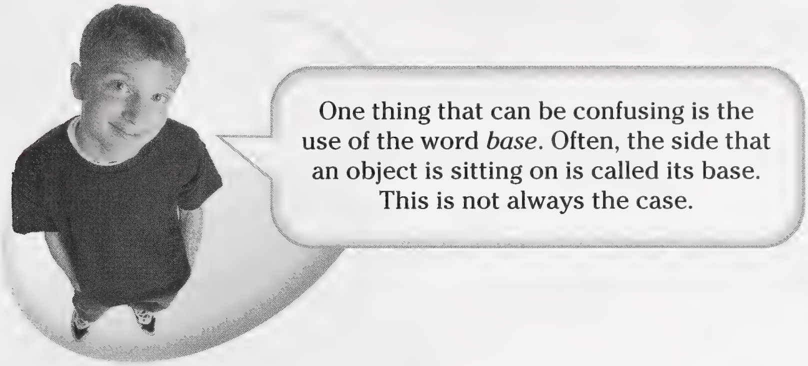


Example

Is the shape to the left a pyramid or a prism?

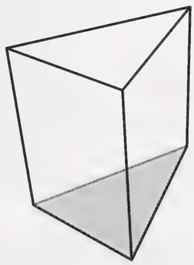
Notice that one end is a point and the other is a ten-sided polygon. The other sides are triangles. These facts tell you that the shape is a pyramid.

Now look at the example on page 313 in your textbook. This example shows how some prisms and pyramids get their names. You should spend some time looking at the reminder on the left side at the bottom of the page. The prefixes listed there show up very regularly when describing shapes.



Look at the triangular prism at the top of page 313. The description says the base and top are congruent triangles. Yet, the picture shows the prism sitting on a rectangular side and the triangle is at the front, facing you. This view is often used to make the shape of the base and top obvious.

The prism could also have been positioned as shown here, where it is sitting on one of its triangular bases.



2. Turn to page 314 in your textbook. Answer question 1 of "Put into Practice."

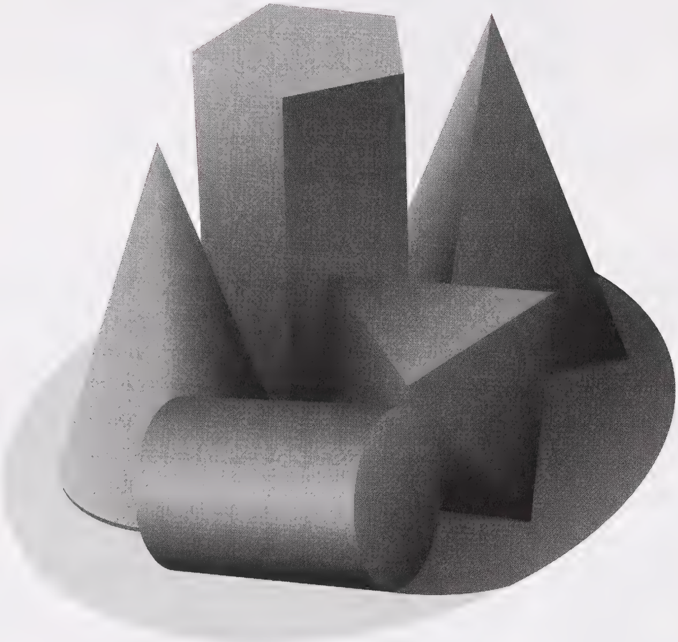
Check your answers on pages 33 and 34 in the Appendix.

Turn to the Section 1 Assignment in Assignment Booklet 6A. Answer question 1.

LESSON 2

Nets and Skeletons

In this lesson you will learn about describing and building three-dimensional objects using two-dimensional shapes.



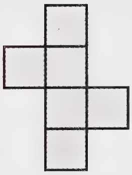
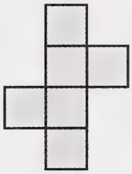
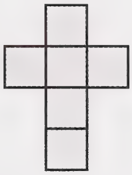
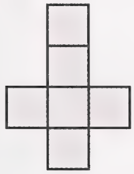
The objects above are quite different from each other. Yet, all of them except for one can be built using the same few shapes—triangles, circles, pentagons, and rectangles. Which one do you think would be the exception? The cone would cause the trouble—you'll see why as you work through this lesson.

To help you visualize how nets and solids relate, use the multimedia segment "Converting from Nets to Solids and Solids to Nets" on the multimedia CD.

Turn to page 315 in your textbook and check out the "get thinking" box at the top of the page. You might think about folding chairs, tents, and steel-frame buildings as examples of places where **skeletons** are used.

You can use the grid paper in the Appendix to simplify your work in this lesson.

1. Turn to page 315 in your textbook and complete "Investigation 1: How many nets?" You will have to decide if **nets** like

 and  **OR**  and 

will count as one net or two nets when you count the number of nets that make a cube.

The first pair of nets represents the same shape, where the second net is the same as the first net, only flipped over. The second pair of nets represents the same shape, where the second net is the same as the first net, but has been rotated 180°.

Check your answers on pages 34 and 35 in the Appendix.

Turn to page 316 in your textbook. Work through Example 1. Notice how the net was created. Imagine cutting the box apart and flattening it. You will need to use this technique for the next few questions.

2. Turn to pages 315 to 317 in your textbook and answer questions 1 to 5 of "Put into Practice 1." While you are working on these questions, you might want to copy the nets onto a sheet of paper, cut them out, and assemble them. Sometimes this is the quickest way to see what objects the nets represent.

Check your answers on pages 35 and 36 in the Appendix.

Using a net is one way to look at a **3-D** object on a 2-D page. Turn to page 318 in your textbook and read the "reminder" box at the top of the page. It suggests another way of simplifying a 3-D object. A skeleton consists of just the edges and vertices (the places where the edges meet) of an object.

3. Turn to page 318 in your textbook and build the objects requested in "Investigation 2: Toothpick Construction." (You can also use bits of playdough or thawed frozen cookie dough for this investigation.) This is a chance for you to be imaginative and let your creativity work for you. This is a good time to think "outside the box."

Check your answers on pages 37 and 38 in the Appendix.

Another way to build skeletons is to run a piece of elastic or string through drinking straws. The vertices are not as clearly marked, but the edges are quite clear. You find this kind of construction used in the frames of dome tents. It allows the frame to be small, light, and easy to pack.

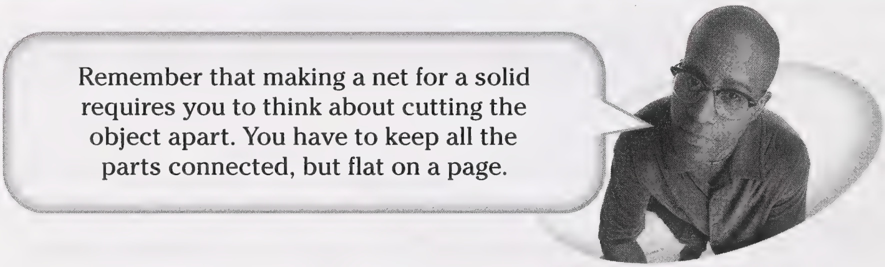
4. Turn to pages 318 and 319 in your textbook and answer questions 8, 9, 10, and 12 of "Put into Practice 2."

Check your answers on pages 38 to 40 in the Appendix.

The objects you've been working with so far in this lesson have all had straight sides and edges. Now it's time to look at a couple of shapes with curved sides and edges, the **cone** and the **cylinder**.

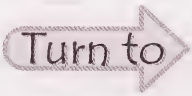
5. Turn to page 319 in your textbook. Use the nets from the Appendix to build the objects requested in "Investigation 3: Cones and Cylinders."

Check your answers on page 40 in the Appendix.



6. Turn to pages 320 and 321 in your textbook and answer questions 13 to 18 of "Put into Practice 3."

Check your answers on pages 40 and 41 in the Appendix.



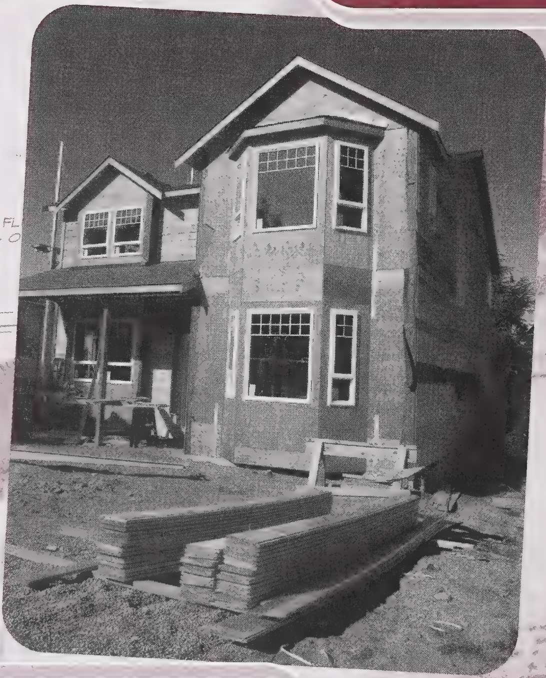
the Section 1 Assignment in Assignment Booklet 6A. Answer question 2.

CONCLUSION

In this section you built cones, cylinders, tetrahedrons, and other pyramids and prisms from nets. You saw how to create nets and skeletons for various 3-D objects.

People have used skeletons and nets for centuries. You can see this in their buildings and tools. Take a close look at the tents the prairie First Nations peoples used. You can see the skeleton of wooden poles supporting the net of animal skin, or, more recently, canvas. Take a look at a tent used for camping. You can also see the skeleton and the net of material covering it.

SECTION 2



Drawings and Design

Planning is of major importance whether you are just planning a house or actually building it. Any little detail can make the difference between failure and success. If the planning stage wasn't done carefully and the drawings made just as carefully, you can imagine what kind of result the builders would have to turn over to the new owner.

Would you want to be the designer who forgot to put the front door on the drawings? How about labelling everything in centimetres instead of inches? Would it be any better if the builders couldn't understand the diagrams you drew?

In this section you will learn about drawing objects from different views so they are more easily understood. You will see the usefulness of top, right, front, and left views of 3-D objects. You will also work at designing shapes and components that fit together well and are pleasing to the eye.

LESSON 1

Plans and Elevations

This lesson is about different ways of drawing 3-D objects on 2-D paper.

This photo makes it look like the tower is about to fall on the statue. Do you think that is really about to happen? In reality, this tower is leaning, but a lot of effort has gone into keeping it from falling over.

What do you think the original plans for the Tower of Pisa would have looked like? Do you think they included drawings that showed the tower from different sides? What other kind of drawings do you think would have made building it possible?

The architect of the tower didn't mean for it to lean—that wasn't part of his plans. The leaning part came after the plans and has helped make this one of the most visited spots on the tourist trail of Europe.

If you have access to the Internet and would like to learn more about the Leaning Tower of Pisa, visit the following website:

http://torre.duomo.pisa.it/towersposters/english_version/

To complete Lesson 1, you must refer to the PDF file called "Module 6: Section 2: Lesson 1 and Related Diagrams" on the multimedia CD. The answers for Lesson 1 are also included in the PDF document.

LESSON 2

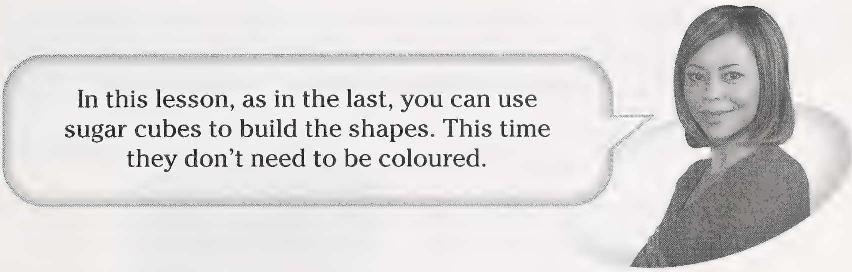
Working with Surface Area

In this lesson you will find the surface area and volume of various shapes.

Before Erica began to paint the living room, there were a lot of decisions to be made, such as the colour and type of paint to use. She also had to figure out where to move the furniture to while the room was being prepped and painted.

Erica's mom calculated the area to be painted in each colour and helped figure out how much of each colour of paint to buy. When Erica got to the paint store, all that was needed was to find out how many cans of paint to buy to cover the area that her mom had calculated. This is a practical use of finding surface area. If you are going to paint something, you have to know how big it is.

Turn to page 327 in your textbook and look at the "get thinking" box at the top of the page. After you have come up with a couple of ideas, you will be ready to continue.



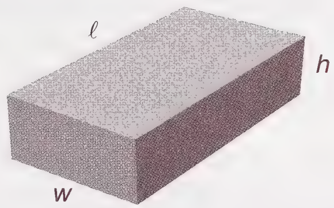
1. Turn to page 327 in your textbook. Work through questions 1 and 2 of "Investigation: Looking at Surface Area." The formulas given on the left side of the page will be useful.

Check your answers on pages 42 to 44 in the Appendix.

Turn to page 328 in your textbook and read the "closer look" box at the top of the page. It notes that design is about more than good looks. The associated costs have to be considered as well.

Continue by studying Example 1 on page 329 in your textbook. It shows you how to find the **surface area** of a rectangular prism. You'll notice that there are six different parts to the surface area calculation for a rectangular prism. You'll also see that the surfaces come in pairs, so only three calculations are needed.

For a rectangular prism, there are three dimensions—the length (ℓ), the width (w), and the height (h).



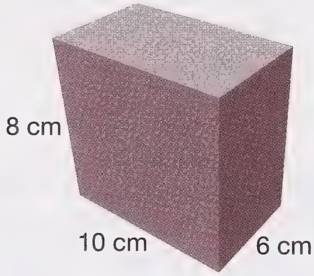
The area of each face is a product of two of these dimensions.

base and top: $A = \ell \times w$
front and back: $A = w \times h$
left and right: $A = h \times \ell$

Example 1 in the textbook also shows how to find the volume of a container.

Example

What is the **volume** and surface area of this rectangular container?



The container has dimensions of 10 cm by 6 cm by 8 cm.

The volume is the product of these three values.

$$\begin{aligned} V &= \ell \times w \times h \\ &= 10\text{ cm} \times 6\text{ cm} \times 8\text{ cm} \\ &= 480\text{ cm}^3 \end{aligned}$$

The volume of this container is 480 cm^3.

To find the surface area, there are three calculations to be carried out.

Base and Top

$$\begin{aligned} A &= \ell \times w \\ &= 10\text{ cm} \times 6\text{ cm} \\ &= 60\text{ cm}^2 \end{aligned}$$

Front and Back

$$\begin{aligned} A &= h \times \ell \\ &= 8\text{ cm} \times 10\text{ cm} \\ &= 80\text{ cm}^2 \end{aligned}$$

Left and Right

$$\begin{aligned} A &= w \times h \\ &= 6\text{ cm} \times 8\text{ cm} \\ &= 48\text{ cm}^2 \end{aligned}$$

The surface area is the sum of the areas of the faces.

$$\begin{aligned} & A_{\text{top}} + A_{\text{bottom}} + A_{\text{front}} + A_{\text{back}} + A_{\text{left}} + A_{\text{right}} \\ &= 60\text{ cm}^2 + 60\text{ cm}^2 + 80\text{ cm}^2 + 80\text{ cm}^2 + 48\text{ cm}^2 + 48\text{ cm}^2 \\ &= 2\left(60\text{ cm}^2 + 80\text{ cm}^2 + 48\text{ cm}^2\right) \\ &= 2 \times 188\text{ cm}^2 \\ &= 376\text{ cm}^2 \end{aligned}$$

Since the top and bottom have the same area, the front and back have the same area, and the left and right have the same area, you could also write the calculation as follows:

$$\begin{aligned}\text{surface area} &= 2\left(A_{\text{top}} + A_{\text{front}} + A_{\text{left}}\right)\\ &= 2\left(60\text{ cm}^2 + 80\text{ cm}^2 + 48\text{ cm}^2\right)\\ &= 2 \times 188\text{ cm}^2\\ &= 376\text{ cm}^2\end{aligned}$$

Some people like to put everything together into one calculation. They'd write it as follows:

$$\begin{aligned}\text{surface area} &= 2\left(A_{\text{top}} + A_{\text{front}} + A_{\text{left}}\right)\\ &= 2\left[(\ell \times w) + (h \times \ell) + (w \times h)\right]\\ &= 2\left[(10\text{ cm} \times 6\text{ cm}) + (8\text{ cm} \times 10\text{ cm}) + (6\text{ cm} \times 8\text{ cm})\right]\\ &= 2\left(60\text{ cm}^2 + 80\text{ cm}^2 + 48\text{ cm}^2\right)\\ &= 2 \times 188\text{ cm}^2\\ &= 376\text{ cm}^2\end{aligned}$$

You have to decide for yourself which method is the easiest for you to use.

2. Turn to pages 328 to 332 in your textbook. Answer questions 1, 3, 5, 6, 8, 9, and 10 of "Put into Practice."

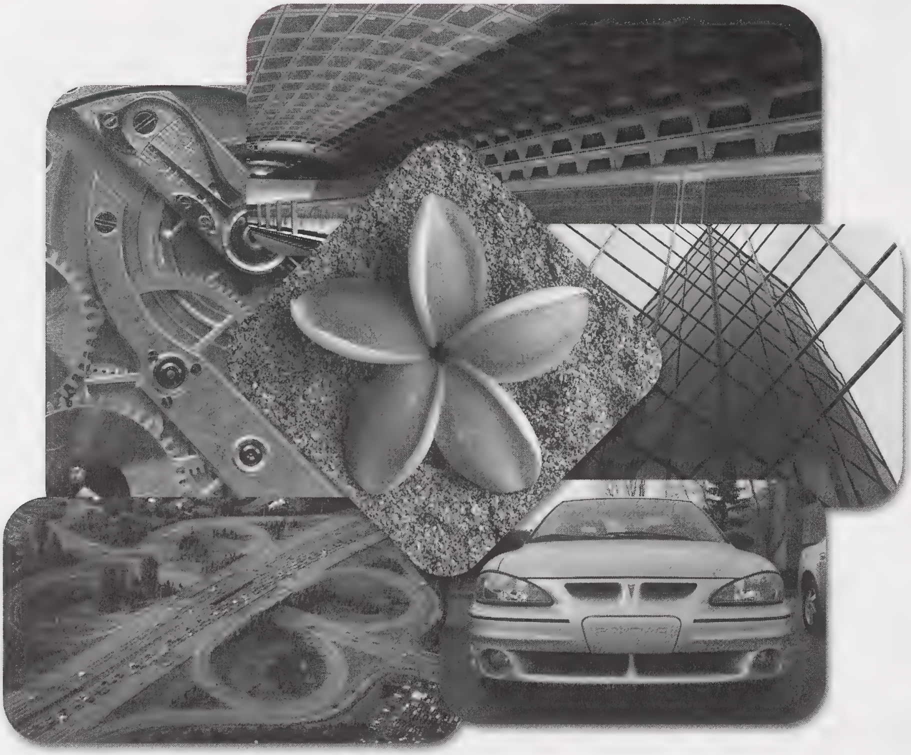
Check your answers on pages 45 to 55 in the Appendix.

Turn to the Section 2 Assignment in Assignment Booklet 6B. Answer question 2.

LESSON 3

Design Problems

This lesson is about designing objects using lines, polygons, arcs, and circles.



When a work of art or a structure catches your eye or a piece of equipment works really well, it's probably because it is well-designed. Good-looking cars, appliances, and art share a lot of common features. Lurking behind the features is the thought and design that went into the product.

When you look at a beautiful flower, what are you likely to see? It will probably be circular and colourful. It will likely be symmetrical. Usually it will have many petals that are all very similar to each other. Keep these ideas in mind when you are designing things in this lesson.

1. Turn to page 333 in your textbook. Work through “Investigation 1: Designing a Notice Board.”

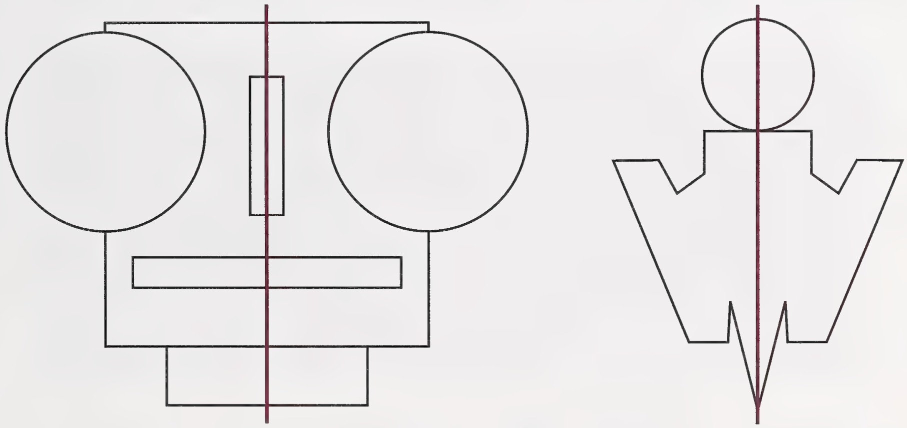
Check your answers on page 56 in the Appendix.

2. Turn to pages 334 to 336 in your textbook. Answer questions 2, 3, 5, and 6 of “Put into Practice.” The formulas in the “reminder” box to the left of question 2 will be useful for several of these questions.

Check your answers on pages 57 to 62 in the Appendix.

The next investigation looks at ways objects are similar to themselves. People, for example, are basically right-left similar. There is one eye, one ear, half of a nose, and half of a mouth on the average right side of a face. The same is true for the average left side of the face. Some scientists claim that the more similar the two sides of a person’s face are, the more attractive the person is to others. This left-right similarity is a form of line symmetry.

Following are some examples of line symmetry. The red lines divide the objects into two mirror images.



When you can fold an image onto itself and the two parts match exactly, the fold line is a line of symmetry. Suppose you cut the image along the line and threw one part away. You could “see” the whole object again by placing a mirror along the cut line. The two parts are mirror images of each other.

3. Turn to page 337 in your textbook. Work through questions a. to c. of “Investigation 2: Hub Cap Design.”

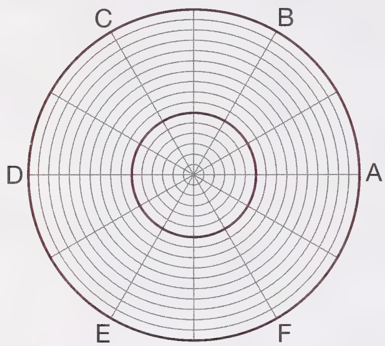
Check your answers on page 62 in the Appendix.

Continue on page 338 by studying the “closer look” box. You will be making a star design. You will need a compass to draw two concentric circles. You may also need a protractor to measure angles. You will use this method to answer the questions at the end of the lesson.

Example

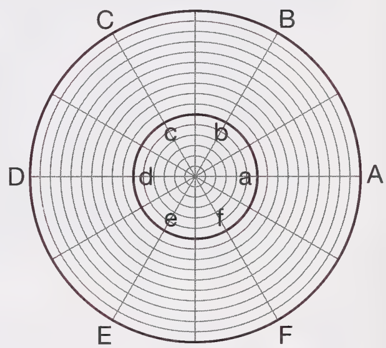
Make a star with six points.

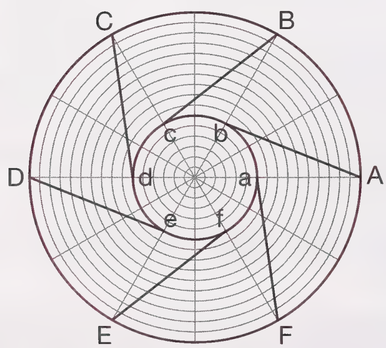
To make this star, you will use circular grid paper with angles of 30° between the rays. Remember that there are 360° in one rotation around the circle. Since six points are needed, divide 360° by 6. You will see that the points will be 60° apart.



The division into 60° segments is easy. Just use every second ray because 30° + 30° = 60°. Name every second ray so they are easy to talk about. The diagram at the left has points named A, B, C, D, E, and F. You will be using the largest circle and the sixth circle from the centre. They are shown with coloured circles in the diagram.

Now label some more points, this time on the smaller of the two red circles. Choose points on the same rays as before, but on the small circle. Label them with the lower case letters a, b, c, d, e, and f.



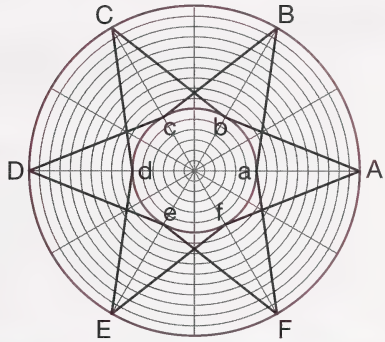


Now join the following points:

- A and b
- C and d
- E and f
- B and c
- D and e
- F and a

Now join some more points to form the star.

- a and B
- c and D
- e and F
- b and C
- d and E
- f and A



Would the design be different if you had chosen the point at the left to be A? No, the result would be the same. This is an example of **rotational symmetry**. (The design would be rotated 180°.)

Example

A circular grid has rays every 10°. If this grid was used to make stars, how many points could the stars have?

There are 360° in one rotation around the circle. The grid divides this into 36 parts; therefore, the possible number of points for the stars would be factors of 36.

The following chart summarizes the number of points that could be made on a star with the factors of 36.

Factors of 36	Number of Points
36 (and 1)	36 points
18 (and 2)	18 points (or 2 points)
12 (and 3)	12 points (or 3 points)
9 (and 4)	9 points (or 4 points)
6 (and 6)	6 points

Remove the pages of circular grid paper from the Appendix. Use them to answer the following questions.

4. Turn to page 339 in your textbook. Answer questions 8 and 9 of “Put into Practice 2.”

Check your answers on page 63 in the Appendix.

Turn to the Section 2 Assignment in Assignment Booklet 6B. Answer question 3.

CONCLUSION

In this section you learned about drawing 3-D objects from different views—the top, right, front, and left views. You learned that these views can also be called the plan view, the right elevation, the front elevation, and the left elevation. You also designed shapes and objects like stars and hubcaps.

Whether you are designing hubcaps, sandcastles, or a real castle, you have to make your ideas clear and easy for the people who will build it to understand.

The people who built the castle and the kids at the beach who built the sandcastle all had a vision for their finished product. Somehow they communicated their vision so the building could be completed.

Have you ever wondered how builders in ancient times described the plans for their buildings? They didn't have computer-assisted drafting tools like architects do today.

MODULE SUMMARY

In this module you learned about designing and constructing various 3-D objects.

In Section 1 you learned about prisms, pyramids, nets, and skeletons. You used nets to build shapes and you figured out what the nets for given shapes would look like. You discovered that you needed to use your imagination to figure out what a net for a given shape would look like.

In Section 2 you learned about plans and elevations, surface area and volume, and making designs to solve problems. You used different views (elevations) to describe and create 3-D objects. You finished by using geometrical shapes to make interesting and useful designs.

Completing a large modern building project requires careful planning. The architects and engineers take painstaking care to make sure the blueprints describe their ideas exactly. The construction companies then follow the blueprints to the letter so the building is strong and stable for years to come.

Turn to Assignment Booklet 6B and complete the Final Module Assignment.

REVIEW

This Review will help you apply what you learned in Module 6 and prepare for the Final Test. Read the skills checklist for this module. Use this list to guide your study and to help you decide how much of the Review you should complete.

Skills Checklist

- ❑ Distinguish a pyramid from a prism.
- ❑ Name pyramids and prisms using the Greek prefixes *tetra*, *penta*, *hexa*, *septa*, *octa*, *nona*, and *deca*.
- ❑ Build nets for prisms, pyramids, and other shapes.
- ❑ Build skeletons for prisms.
- ❑ Distinguish a cone from a cylinder.
- ❑ Build nets for cones and cylinders.
- ❑ Draw front, top, left, and right views of given 3-D shapes.
- ❑ Use views of 3-D objects to make possible objects.
- ❑ Find surface areas of prisms and pyramids.
- ❑ Find volumes of cylinders and prisms and shapes made of these objects.
- ❑ Design objects that have a line of symmetry.
- ❑ Design objects that have rotational symmetry.
- ❑ Design objects given restrictions on placement of sub-objects.

Review Questions

Turn to pages 340 to 343 in your textbook. Answer "Review of Unit Six" questions 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, and 14

Check your answers on pages 63 to 71 in the Appendix.

Congratulations!

You have finished all six modules of

MATHEMATICS 24

Module 1 INDEPENDENT LIVING

Module 2 WHEELS

Module 3 APPLYING GEOMETRY

Module 4 MAPS, DATA, and PROBABILITY

Module 5 STATISTICS

Module 6 DESIGN and CONSTRUCTION

Hopefully you found Mathematics 24 to be challenging and rewarding! Best wishes for future success in applying your new skills!

MATHEMATICS 24

Appendix

GLOSSARY
ANSWER KEY
IMAGE CREDITS
LEARNING AIDS

Glossary

3-D: having three dimensions

Objects like boxes, cars, and people are 3-D objects.

cone: a 3-D figure with a circular base and a curved surface that tapers to a point

cylinder: a 3-D figure with two parallel congruent circular faces

front elevation: a view of an object directly from the front

front view: a view of an object directly from the front

left elevation: a view of an object directly from the left

left view: a view of an object directly from the left

line of symmetry: a line that divides an object into two mirror images

net: a pattern that can be folded to make a 3-D figure

It shows the surface area of the figure.

plan view: a view of an object directly from the top

prism: a 3-D figure with two parallel and congruent polygon bases and with rectangular faces

pyramid: a 3-D solid with a polygon base and other faces that are triangles that meet at a vertex

right elevation: a view of an object directly from the right

right view: a view of an object directly from the right

rotational symmetry: a property of an object

An object has rotational symmetry if it looks identical after being rotated through some angle less than 360°.

skeleton: a 3-D structure containing only the edges and vertices of a 3-D solid

surface area: the sum of the areas of the faces of a 3-D object

top view: a view of an object directly from the top

volume: a measure of the space occupied by a solid

Answer Key

Section 1: Prisms and Nets

Lesson 1: Prisms and Pyramids

1. **Textbook, page 312, "Investigation: Prisms or Pyramids?", questions 1 to 3**

   1. **a.** prism: There is no pointed top and the sides are rectangles.
      b. prism: There is no pointed top and the sides are rectangles.
      c. pyramid: There is a pointed top and the sides are triangles.
      d. prism: There is no pointed top and the sides are rectangles.
      e. prism: There is no pointed top and the sides are rectangles.
      f. prism: There is no pointed top and the sides are rectangles.
      g. pyramid: There is a pointed top and the sides are triangles.

   2. Your list of prisms might include the following:

      - paperback books
      - geometry sets
      - cupboards
      - boxes of cereal
      - CD cases
      - packages of paper
      - flat-roofed buildings
      - bunkhouse trailers
      - stereo speakers
      - footstools
      - toasters

      Your list of pyramids might include the following:

      - buildings (like the Muttart Conservatory and City Hall in Edmonton)
      - four-sided dice
      - stacks of goods in store displays

   3. A prism has two ends that are the same (congruent) and sides made of rectangles.

      A pyramid has some shape (like a triangle, square, or octagon) as the bottom and a point at the top. The sides are triangles.

2. **Textbook, page 314, "Put into Practice," question 1**

   1.

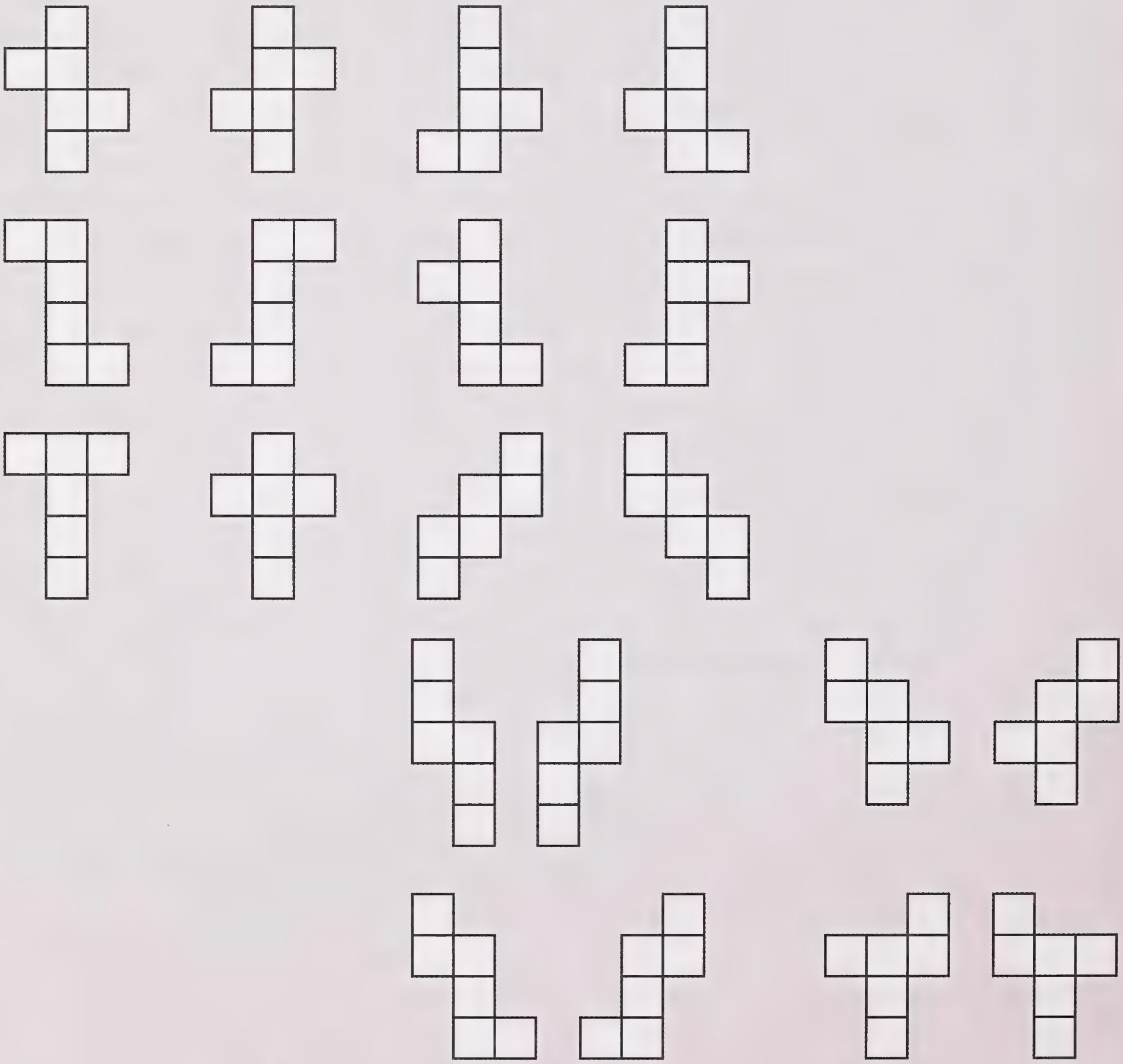
	Name	Description of Base	Description of Top	Description of Faces
a.	hexagonal pyramid	hexagon	point	triangles
b.	triangular prism	triangle	triangle	rectangles
c.	rectangular prism	rectangle	rectangle	rectangles

d.	pentagonal pyramid	pentagon	point	triangles
e.	pentagonal prism	pentagon	pentagon	rectangles
f.	hexagonal prism	hexagon	hexagon	rectangles
g.	rectangular pyramid	rectangle	point	triangles

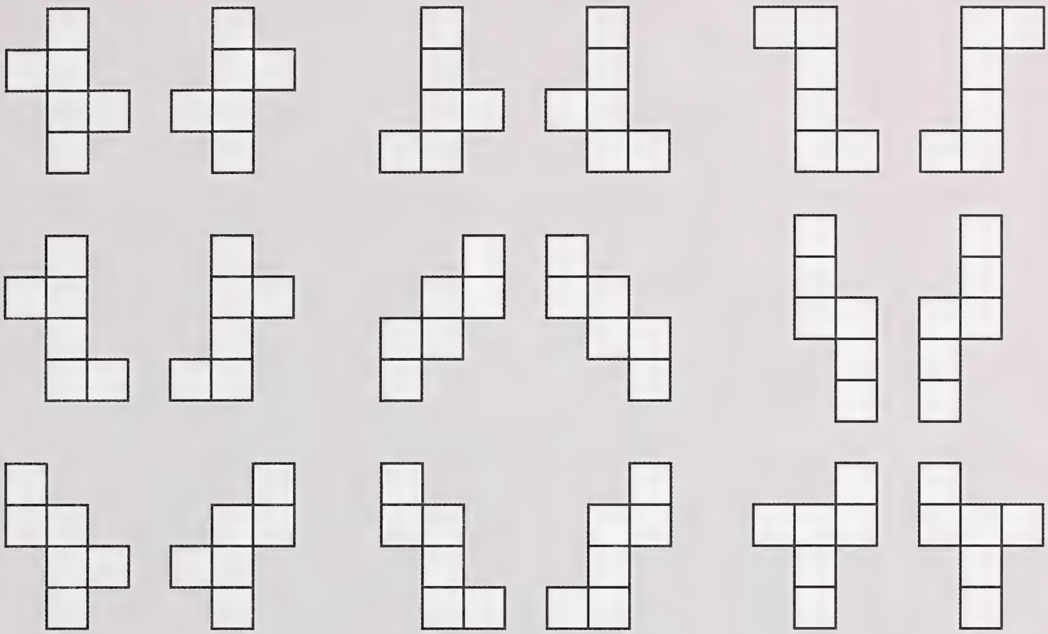
Lesson 2: Nets and Skeletons

1. Textbook, page 315, "Investigation 1: How many nets?"

The given net will let you create a cube. The following nets all make a cube.



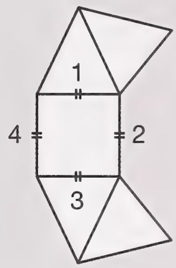
You may see that several of these nets are related by flips.



There are 11 different nets if you count flips and rotations as only one net.

2. Textbook, pages 315 to 317, "Put into Practice 1," questions 1 to 5

1. Net a. can be used to build a square pyramid.



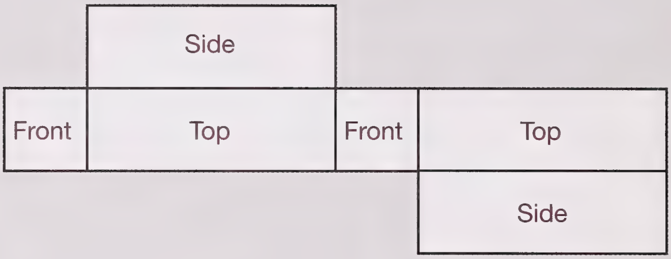
Net b. cannot be used to build a square pyramid. There is no triangle to rest on side 4 of the base. (See the diagram at the right.)

Net c. can be used to build a square pyramid.

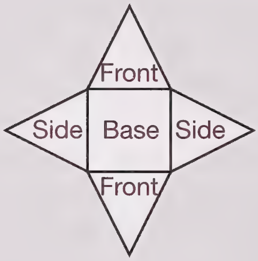
2. Net a. can be used to build a tetrahedron.
Net b. cannot be used to build a tetrahedron.
Net c. can be used to build a tetrahedron.

3. a. The object requires two rectangular fronts, two rectangular tops, and two rectangular sides.

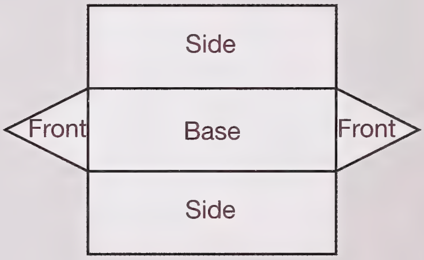
b. The object requires two rectangular fronts, two rectangular tops, and two rectangular sides.



c. The object requires a rectangular base, two triangular fronts, and two triangular sides.



d. The object requires two triangular fronts, two rectangular sides, and a rectangular base.

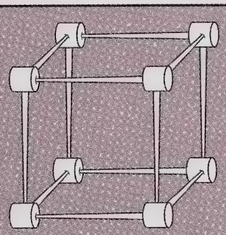
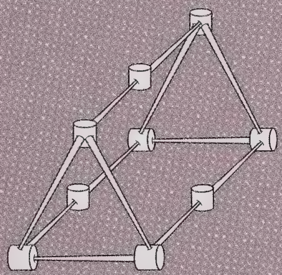
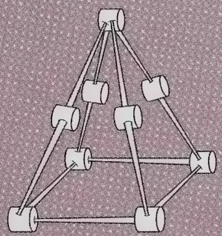
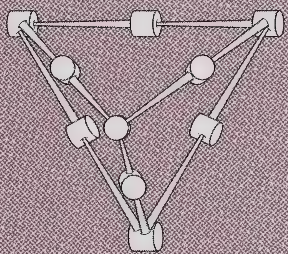
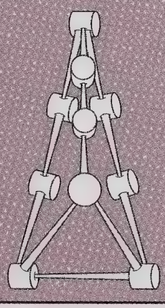


4. The completed prism is your check.

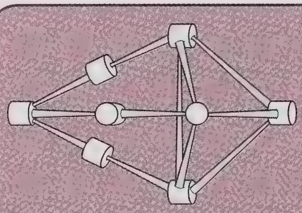
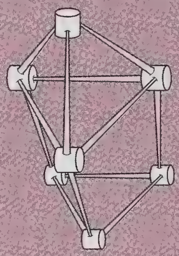
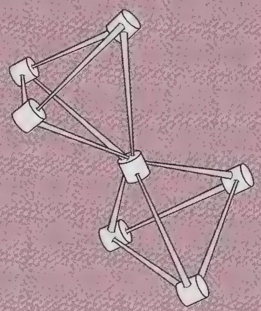
5. The completed pyramid is your check.

3. Textbook, page 318, "Investigation 2: Toothpick Construction"

The following five objects are given as answers in the textbook.

	cube
	triangular prism with rectangular faces measuring two toothpicks by one toothpick
	square pyramid with rising sides measuring two toothpicks
	tetrahedron with each side measuring two toothpicks
	tetrahedron with rising sides measuring three toothpicks

You might have built other shapes like the following:

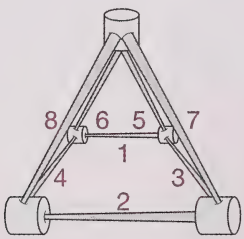
	hexahedron with one set of edges that are twice the length of the other edges
	triangular prism with a pyramid on top
	two joined tetrahedra

There are many more shapes you could have created.

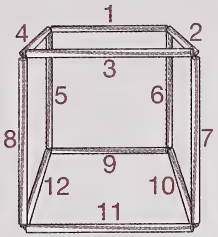
4. Textbook, pages 318 to 319, "Put into Practice 2," questions 8, 9, 10, and 12

8. a. Emma built a square pyramid.

b. Emma would need eight toothpicks. The toothpicks are numbered in the diagram below.

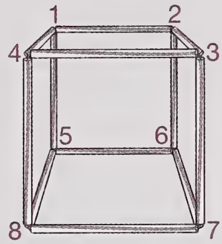


9. a.



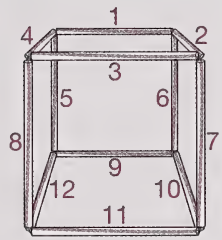
Jack and Gerry used 12 straws. They are numbered in the diagram to the left.

b.



There are eight vertices. They are numbered in the diagram to the left.

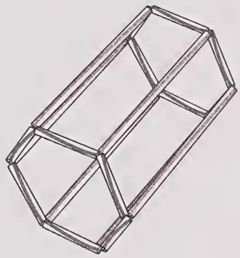
c.



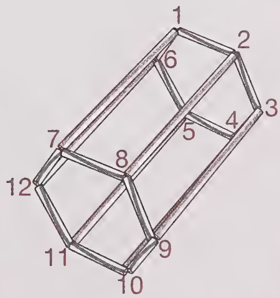
There are 12 edges. They are numbered in the diagram to the left.

10. a. The diagram in the textbook suggests that two different lengths of straws are needed. One length is used for the sides of the two hexagons and a different length is used between the two hexagons.

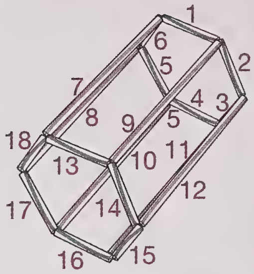
b. Amy and Shelina used six long straws and 12 short straws. The short straws are shown in grey and the long straws are shown in red in the diagram to the right.



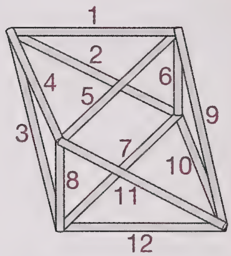
c. The hexagonal prism has 12 vertices. They are numbered in the diagram to the right.



d. The prism has 18 edges. They are numbered in the diagram to the right.



12. a. The frame requires 12 rods. They are numbered in the diagram to the right.



b. There are 12 rods, each measuring 75 cm long. The total length of material will be 12×75 cm = 900 cm. This is 9 m of material.

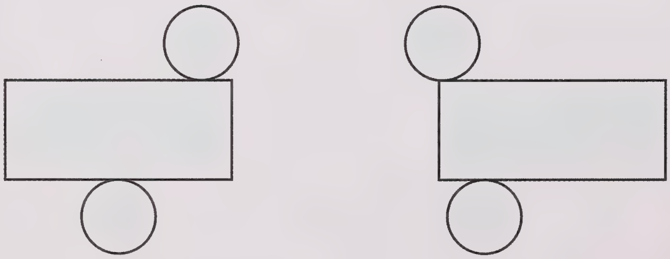
c. Nine metres of material will cost $9 \times \$2.58 = \$23.22$ before tax.

5. **Textbook, page 319, "Investigation 3: Cones and Cylinders"**

One net lets you construct a cone. The other net lets you construct a cylinder.

6. **Textbook, pages 320 and 321, "Put into Practice 3," questions 13 to 18**

13. The net for the cylinder should consist of a rectangle with a circle on each of the long sides. Two possible nets are shown below.



14. Net 1 and Net 2 both would work for Anila. They both would let her construct a cylinder.

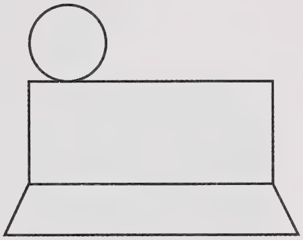
Net 3 would not work. One end of the cylinder would be open and one end would have two covers.

15. a. This net is for a cone. You may have thought of objects like an ice cream cone, a pylon used at road construction sites, or a light fixture.

b. These objects have a round base and taper to a point.

16. The conical part of the cone could be built from a net like the one shown to the right.

17. The circle and the rectangle below it form the cylindrical part of the hat. The brim by itself would be a circle with a circular hole in it. When you cut this circle and leave it attached to the rectangle, the net shown to the right is created.



18. The awning has sides that are curved and a rectangular piece joining them. Two possible nets are shown.

Section 2: Drawings and Design

Lesson 1: Plans and Elevations

The answers for Lesson 1 are included in the PDF document called "Module 6: Section 2: Lesson 1 and Related Diagrams" on the multimedia CD.

Lesson 2: Working with Surface Area

1. Textbook, page 327, "Investigation: Looking at Surface Area," questions 1 and 2

1. a. Using the formula in the reminder, the volume is found as follows.

$$\begin{aligned} V &= (\text{area of base}) \times \text{height} \\ &= (b \times h) \times \text{height} \\ &= (4 \times 1) \times 3 \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

The volume of the shape is 12 cubic units.

b. The area of the front of the prism is found as follows.

$$\begin{aligned} A &= b \times h \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

The area of the front face is 12 square units.

c. The area of the back of the prism is found as follows.

$$\begin{aligned} A &= b \times h \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

The area of the back face is 12 square units.

d. The area of the top of the prism is found as follows.

$$\begin{aligned} A &= b \times h \\ &= 4 \times 1 \\ &= 4 \end{aligned}$$

The area of the top face is 4 square units.

e. The area of the bottom face is found as follows.

$$\begin{aligned} A &= b \times h \\ &= 4 \times 1 \\ &= 4 \end{aligned}$$

The area of the bottom face is 4 square units.

f. The area of each side of the prism is found as follows.

$$\begin{aligned} A &= b \times h \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

The area of a side face is 3 square units.

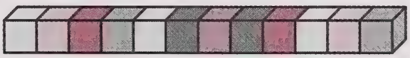
g. The total surface area is 38 square units.

$$\begin{aligned} \text{surface area} &= A_{\text{front}} + A_{\text{back}} + A_{\text{top}} + A_{\text{bottom}} + A_{\text{right}} + A_{\text{left}} \\ &= 12 + 12 + 4 + 4 + 3 + 3 \\ &= 38 \end{aligned}$$

h. The front and back have the same area. The top and bottom have the same area. The two sides have the same area.

2. a. Following are some other prisms that can be made with 12 cubes.

This is a 12-by-1-by-1 prism.

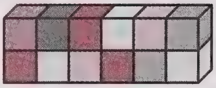


Top and Bottom	**Front and Back**	**Sides**
$A = b \times h$	$A = b \times h$	$A = b \times h$
$= 12 \times 1$	$= 12 \times 1$	$= 1 \times 1$
$= 12$	$= 12$	$= 1$

$$\begin{aligned} \text{surface area} &= A_{\text{top}} + A_{\text{bottom}} + A_{\text{front}} + A_{\text{back}} + A_{\text{right}} + A_{\text{left}} \\ &= 12 + 12 + 12 + 12 + 1 + 1 \\ &= 50 \end{aligned}$$

The surface area is 50 square units.

This is a 6-by-2-by-1 prism.

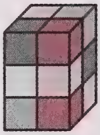


Top and Bottom	Front and Back	Sides
$A = b \times h$	$A = b \times h$	$A = b \times h$
$= 6 \times 1$	$= 6 \times 2$	$= 1 \times 2$
$= 6$	$= 12$	$= 2$

$$\begin{aligned} \text{surface area} &= A_{top} + A_{bottom} + A_{front} + A_{back} + A_{right} + A_{left} \\ &= 6 + 6 + 12 + 12 + 2 + 2 \\ &= 40 \end{aligned}$$

The surface area is 40 square units.

This is a 2-by-2-by-3 prism.



Top and Bottom	Front and Back	Sides
$A = b \times h$	$A = b \times h$	$A = b \times h$
$= 2 \times 2$	$= 2 \times 3$	$= 2 \times 3$
$= 4$	$= 6$	$= 6$

$$\begin{aligned} \text{surface area} &= A_{top} + A_{bottom} + A_{front} + A_{back} + A_{right} + A_{left} \\ &= 4 + 4 + 6 + 6 + 6 + 6 \\ &= 32 \end{aligned}$$

The surface area is 32 square units.

b. The 2-by-2-by-3 prism had the least surface area.

2. Textbook, pages 328 to 332, "Put into Practice," questions 1, 3, 5, 6, 8, 9, and 10

1. a. and **b.** Following are four prisms made of 16 cubes.

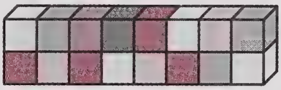
This is a 16-by-1-by-1 prism.

Top and Bottom	**Front and Back**	**Sides**
$A = b \times h$	$A = b \times h$	$A = b \times h$
$= 16 \times 1$	$= 16 \times 1$	$= 1 \times 1$
$= 16$	$= 16$	$= 1$

$$\begin{aligned}\text{surface area} &= A_{top} + A_{bottom} + A_{front} + A_{back} + A_{right} + A_{left} \\ &= 16 + 16 + 16 + 16 + 1 + 1 \\ &= 66\end{aligned}$$

The surface area is 66 square units.

This is an 8-by-2-by-1 prism.

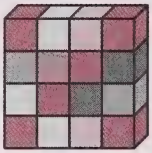


Top and Bottom	**Front and Back**	**Sides**
$A = b \times h$	$A = b \times h$	$A = b \times h$
$= 8 \times 1$	$= 8 \times 2$	$= 1 \times 2$
$= 8$	$= 16$	$= 2$

$$\begin{aligned}\text{surface area} &= A_{top} + A_{bottom} + A_{front} + A_{back} + A_{right} + A_{left} \\ &= 8 + 8 + 16 + 16 + 2 + 2 \\ &= 52\end{aligned}$$

The surface area is 52 square units.

This is a 4-by-4-by-1 prism.



Top and Bottom

$$\begin{aligned} A &= b \times h \\ &= 4 \times 1 \\ &= 4 \end{aligned}$$

Front and Back

$$\begin{aligned} A &= b \times h \\ &= 4 \times 4 \\ &= 16 \end{aligned}$$

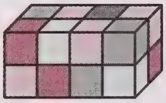
Sides

$$\begin{aligned} A &= b \times h \\ &= 1 \times 4 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{surface area} &= A_{\text{top}} + A_{\text{bottom}} + A_{\text{front}} + A_{\text{back}} + A_{\text{right}} + A_{\text{left}} \\ &= 4 + 4 + 16 + 16 + 4 + 4 \\ &= 48 \end{aligned}$$

The surface area is 48 square units.

This is a 4-by-2-by-2 prism.



Top and Bottom

$$\begin{aligned} A &= b \times h \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

Front and Back

$$\begin{aligned} A &= b \times h \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

Sides

$$\begin{aligned} A &= b \times h \\ &= 2 \times 2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{surface area} &= A_{\text{top}} + A_{\text{bottom}} + A_{\text{front}} + A_{\text{back}} + A_{\text{right}} + A_{\text{left}} \\ &= 8 + 8 + 8 + 8 + 4 + 4 \\ &= 40 \end{aligned}$$

The surface area is 40 square units.

c. The 4-by-2-by-2 prism has the least surface area.

3. **a.** The volume of the box is found as follows.

$$\begin{aligned} V &= \ell \times w \times h \\ &= 15\text{ cm} \times 9\text{ cm} \times 5\text{ cm} \\ &= 675\text{ cm}^3 \end{aligned}$$

The box's volume is 675 cm^3.

b. The surface area is found as follows.

Top and Bottom	**Front and Back**	**Sides**
$A = b \times h (\text{or } \ell \times w)$	$A = b \times h (\text{or } \ell \times h)$	$A = b \times h (\text{or } w \times h)$
$= 15\text{ cm} \times 9\text{ cm}$	$= 15\text{ cm} \times 5\text{ cm}$	$= 9\text{ cm} \times 5\text{ cm}$
$= 135\text{ cm}^2$	$= 75\text{ cm}^2$	$= 45\text{ cm}^2$

$$\begin{aligned} \text{surface area} &= A_{\text{top}} + A_{\text{bottom}} + A_{\text{front}} + A_{\text{back}} + A_{\text{right}} + A_{\text{left}} \\ &= 135\text{ cm}^2 + 135\text{ cm}^2 + 75\text{ cm}^2 + 75\text{ cm}^2 + 45\text{ cm}^2 + 45\text{ cm}^2 \\ &= 510\text{ cm}^2 \end{aligned}$$

The surface area is 510 cm^2.

5. **a.** The volume is found as follows.

$$\begin{aligned} V &= \ell \times w \times h \\ &= 30\text{ cm} \times 20\text{ cm} \times 8\text{ cm} \\ &= 4800\text{ cm}^3 \end{aligned}$$

The box's volume is 4800 cm^3.

b. The surface area is found as follows.

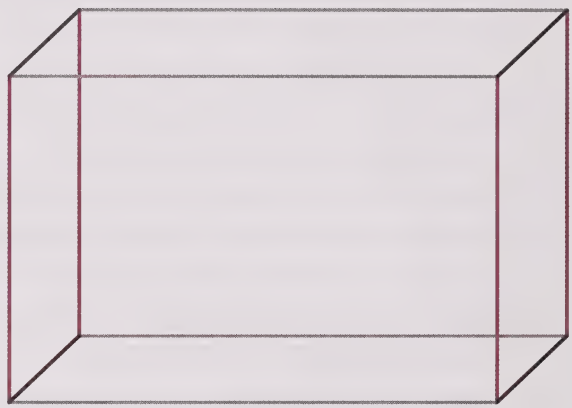
Top and Bottom	**Front and Back**	**Sides**
$A = b \times h (\text{or } \ell \times w)$	$A = b \times h (\text{or } \ell \times h)$	$A = b \times h (\text{or } w \times h)$
$= 30 \text{ cm} \times 20 \text{ cm}$	$= 30 \text{ cm} \times 8 \text{ cm}$	$= 20 \text{ cm} \times 8 \text{ cm}$
$= 600 \text{ cm}^2$	$= 240 \text{ cm}^2$	$= 160 \text{ cm}^2$

$$\begin{aligned} \text{surface area} &= A_{top} + A_{bottom} + A_{front} + A_{back} + A_{right} + A_{left} \\ &= 600 \text{ cm}^2 + 600 \text{ cm}^2 + 240 \text{ cm}^2 + 240 \text{ cm}^2 + 160 \text{ cm}^2 + 160 \text{ cm}^2 \\ &= 2000 \text{ cm}^2 \end{aligned}$$

The surface area is 2000 cm^2.

c. The following diagram is an edge diagram of the box.

Each red side is 20 cm long, each grey side is 30 cm long, and each black side is 8 cm long. There are four red sides, four grey sides, and four black sides.



From the diagram, you can see that the length of tape needed can be found as follows.

$$\begin{aligned} 4 \times (30 + 20 + 8) \text{ cm} &= 4 \times 58 \text{ cm} \\ &= 232 \text{ cm} \end{aligned}$$

6. a. The boxes have the same volume.

$$\begin{aligned} V &= \ell \times w \times h \\ &= 4 \text{ cm} \times 6 \text{ cm} \times 4 \text{ cm} \\ &= 96 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} V &= \ell \times w \times h \\ &= 16 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm} \\ &= 96 \text{ cm}^3 \end{aligned}$$

b. The surface area of the long, thin box is calculated as follows.

Top and Bottom	**Front and Back**	**Sides**
$A = b \times h$	$A = b \times h$	$A = b \times h$
$= 16\text{ cm} \times 2\text{ cm}$	$= 16\text{ cm} \times 3\text{ cm}$	$= 2\text{ cm} \times 3\text{ cm}$
$= 32\text{ cm}^2$	$= 48\text{ cm}^2$	$= 6\text{ cm}^2$

$$\begin{aligned}\text{surface area} &= A_{top} + A_{bottom} + A_{front} + A_{back} + A_{right} + A_{left}\\ &= 32\text{ cm}^2 + 32\text{ cm}^2 + 48\text{ cm}^2 + 48\text{ cm}^2 + 6\text{ cm}^2 + 6\text{ cm}^2\\ &= 172\text{ cm}^2\end{aligned}$$

The surface area of the other box is calculated as follows.

Top and Bottom	**Front and Back**	**Sides**
$A = b \times h$	$A = b \times h$	$A = b \times h$
$= 6\text{ cm} \times 4\text{ cm}$	$= 6\text{ cm} \times 4\text{ cm}$	$= 4\text{ cm} \times 4\text{ cm}$
$= 24\text{ cm}^2$	$= 24\text{ cm}^2$	$= 16\text{ cm}^2$

$$\begin{aligned}\text{surface area} &= A_{top} + A_{bottom} + A_{front} + A_{back} + A_{right} + A_{left}\\ &= 24\text{ cm}^2 + 24\text{ cm}^2 + 24\text{ cm}^2 + 24\text{ cm}^2 + 16\text{ cm}^2 + 16\text{ cm}^2\\ &= 128\text{ cm}^2\end{aligned}$$

The long thin, box has the larger surface area.

c. The difference in surface areas is $172\text{ cm}^2 - 128\text{ cm}^2 = 44\text{ cm}^2$.

d. You could suggest that Ms. Jenkins use the compact 4-by-6-by-4 box because it would use less material and cost less to make.

You could suggest that Ms. Jenkins use the 2-by-3-by-16 box because it has a unique shape that would set itself apart in the stores, and possibly get better sales.

8. For this object, you would start by finding the volume and surface area of a solid block. From the volume of the block, you would have to subtract the volume of the cylinder that is cut out of the block. From the surface area of the object, you would have to subtract the circular areas of the ends of the cylinder and add the surface area of the side of the cylinder.

   a. First, find the volume of the solid block.

$$\begin{aligned} V &= \ell \times w \times h \\ &= 4\text{ cm} \times 10\text{ cm} \times 6\text{ cm} \\ &= 240\text{ cm}^3 \end{aligned}$$

   Second, find the volume of the cylinder that is removed from the block.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi\left(\frac{3}{2}\text{ cm}\right)^2 \times 4\text{ cm} \\ &\doteq 28.274\,333\,88\text{ cm}^3 \end{aligned}$$

   The volume of the given object is the difference of these volumes.

$$240\text{ cm}^3 - 28.274\,333\,88\text{ cm}^3 = 211.725\,666\,1\text{ cm}^3$$

   The volume, to the nearest cubic centimetre, is 212 cm^3.

   b. First, find the surface area of the solid block. (Notice the slight change in the formula for used for the area in each column.)

Top and Bottom	**Front and Back**	**Sides**
$A = \ell \times w$	$A = w \times h$	$A = \ell \times h$
$= 4\text{ cm} \times 10\text{ cm}$	$= 10\text{ cm} \times 6\text{ cm}$	$= 4\text{ cm} \times 6\text{ cm}$
$= 40\text{ cm}^2$	$= 60\text{ cm}^2$	$= 24\text{ cm}^2$

$$\begin{aligned} \text{surface area} &= A_{\text{top}} + A_{\text{bottom}} + A_{\text{front}} + A_{\text{back}} + A_{\text{right}} + A_{\text{left}} \\ &= 40\text{ cm}^2 + 40\text{ cm}^2 + 60\text{ cm}^2 + 60\text{ cm}^2 + 24\text{ cm}^2 + 24\text{ cm}^2 \\ &= 248\text{ cm}^2 \end{aligned}$$

You could also do all of the calculations at once.

$$\begin{aligned}\text{surface area} &= 2\times\left[(\ell\times w)+(w\times h)+(\ell\times h)\right]\\ &= 2\times\left[(4\times 10)+(10\times 6)+(4\times 6)\right]\\ &= 2\times 124\\ &= 248\ \text{cm}^2\end{aligned}$$

Second, find the area of the circles that are cut out from the front and back.

$$\begin{aligned}A &= \pi r^2\\ &= \pi\left(\frac{3}{2}\ \text{cm}\right)^2\\ &\doteq 7.068\,583\,471\end{aligned}$$

Two such areas have to be subtracted.

Third, find the area of the side of the cylinder.

$$\begin{aligned}\text{surface area} &= \pi\times d\times h\\ &= \pi\times 3\ \text{cm}\times 4\ \text{cm}\\ &\doteq 37.699\,111\,84\ \text{cm}^2\end{aligned}$$

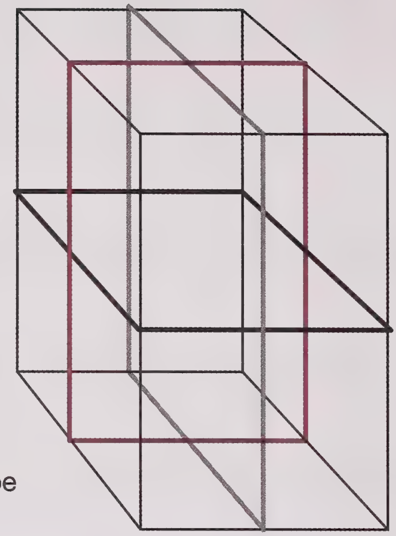
The surface area is the surface area of the block, plus the surface area of the side of the cylinder, less the area of the two circles.

$$248\ \text{cm}^2 + 37.699\,111\,84\ \text{cm}^2 - \left(2\times 7.068\,583\,471\ \text{cm}^2\right) = 271.561\,944\,9\ \text{cm}^2$$

This is 272 cm^2 to the nearest square centimetre.

9. a. The diagram to the right shows the tape on a transparent box.

There are three loops of tape. The grey line shows a piece across the top at 30 cm, a piece down the back at 40 cm, and pieces along the bottom and front at 30 cm and 40 cm. The red line shows a piece 25 cm across the top, 40 cm down the right side, 25 cm across the bottom, and 40 cm up the left side. The black line shows a piece 25 cm across the front, 30 cm across the right side, 25 cm across the back, and 30 cm along the left side.



From the diagram, you can see that the length of tape needed is grey length + red length + black length.

$$\begin{aligned}(30+40+30+40)+(25+40+25+40)+(25+30+25+30) &= 140+130+110\\ &= 380 \text{ cm}\end{aligned}$$

b. 380 cm is the same as 3.8 m. Nineteen boxes would require 19 times as much tape. This would be 19×3.8 m = 72.2 m.

c. The following table shows different ways to buy tape for the boxes. The cheapest way is to buy three 20-m rolls, one 10-m roll, and one 5-m roll. This would cost $15.71.

Tape Lengths	Cost of 5-m Rolls	Cost of 10-m Rolls	Cost of 20-m Rolls	Total Cost
Use only 5-m rolls	15 × $1.32			$19.80
Use only 10-m rolls		8 × $2.24		$17.92
Use only 20-m rolls			4 × $3.87	$15.48
Use a mixture of tapes	1 × $1.32 = $1.32	1 × $2.24 = $2.24	3 × $3.87 = $11.61	$15.17

d. The values in the table and the following facts led to the purchase decision:

- $15 \times 5 = 75$, so it would take a little less than 15 rolls of 5-m tape.
- $8 \times 10 = 80$, so it would take a little less than 8 rolls of 10-m tape.
- $4 \times 20 = 80$, so it would take a little less than 4 rolls of 20-m tape.
- $5 + 10 + 60 = 75$, so 1 roll of 5-m tape, 1 roll of 10-m tape, and 3 rolls of 20-m tape would also work.

To keep the waste to a minimum and the cost lower, a mixture of lengths is the best choice.

10. Use the fact that $48 = 2 \times 2 \times 2 \times 2 \times 3$. Some possible arrangements are $3 \times 2 \times 8$, $3 \times 4 \times 4$, $6 \times 2 \times 4$, $12 \times 2 \times 2$, $24 \times 2 \times 1$, $1 \times 6 \times 8$, and $48 \times 1 \times 1$. The questions done so far show that the most compact packages give the smallest surface area. This will be true for the pop can as well.

Build a spreadsheet that shows all the ways to build a rectangular prism to hold 48 cans of pop. The spreadsheet template Mod_6_1.xlt, in the spreadsheets folder on the multimedia CD, was used to create the spreadsheets that follow. Start by picking the ways of placing the pop cans. Then calculate the dimensions of the container, remembering that each can is 13 cm tall and 7 cm in diameter. Notice that the surface area of a rectangular prism doesn't change if the length and width are interchanged. This makes the following table much shorter.

	A	B	C	D	E	F	G
1	Height (in Cans)	Length (in Cans)	Width (in Cans)	Height (in cm)	Length (in cm)	Height (in cm)	Surface Area (in square cm)
2	1	1	48	=13*A2	=7*B2	=7*C2	=2*(D2*E2+E2*F2+D2*F2)
3	1	2	24	=13*A3	=7*B3	=7*C3	=2*(D3*E3+E3*F3+D3*F3)
4	1	3	16	=13*A4	=7*B4	=7*C4	=2*(D4*E4+E4*F4+D4*F4)
5	1	4	12	=13*A5	=7*B5	=7*C5	=2*(D5*E5+E5*F5+D5*F5)
6	1	6	8	=13*A6	=7*B6	=7*C6	=2*(D6*E6+E6*F6+D6*F6)
7	2	1	24	=13*A7	=7*B7	=7*C7	=2*(D7*E7+E7*F7+D7*F7)
8	2	2	12	=13*A8	=7*B8	=7*C8	=2*(D8*E8+E8*F8+D8*F8)
9	2	3	8	=13*A9	=7*B9	=7*C9	=2*(D9*E9+E9*F9+D9*F9)
10	2	4	6	=13*A10	=7*B10	=7*C10	=2*(D10*E10+E10*F10+D10*F10)
11	3	1	16	=13*A11	=7*B11	=7*C11	=2*(D11*E11+E11*F11+D11*F11)
12	3	2	8	=13*A12	=7*B12	=7*C12	=2*(D12*E12+E12*F12+D12*F12)
13	3	4	4	=13*A13	=7*B13	=7*C13	=2*(D13*E13+E13*F13+D13*F13)
14	4	1	12	=13*A14	=7*B14	=7*C14	=2*(D14*E14+E14*F14+D14*F14)
15	4	2	6	=13*A15	=7*B15	=7*C15	=2*(D15*E15+E15*F15+D15*F15)
16	4	3	4	=13*A16	=7*B16	=7*C16	=2*(D16*E16+E16*F16+D16*F16)

17	6	1	8	=13*A17	=7*B17	=7*C17	=2*(D17*E17+E17*F17+D17*F17)
18	6	2	4	=13*A18	=7*B18	=7*C18	=2*(D18*E18+E18*F18+D18*F18)
19	8	1	6	=13*A19	=7*B19	=7*C19	=2*(D19*E19+E19*F19+D19*F19)
20	8	2	3	=13*A20	=7*B20	=7*C20	=2*(D20*E20+E20*F20+D20*F20)
21	12	1	4	=13*A21	=7*B21	=7*C21	=2*(D21*E21+E21*F21+D21*F21)
22	12	2	2	=13*A22	=7*B22	=7*C22	=2*(D22*E22+E22*F22+D22*F22)
23	24	1	2	=13*A23	=7*B23	=7*C23	=2*(D23*E23+E23*F23+D23*F23)
24	48	1	1	=13*A24	=7*B24	=7*C24	=2*(D24*E24+E24*F24+D24*F24)

The values are shown in the following table.

	A	B	C	D	E	F	G
1	Height (in Cans)	Length (in Cans)	Width (in Cans)	Height (in cm)	Length (in cm)	Height (in cm)	Surface Area (in square cm)
2	1	1	48	13	7	336	13622
3	1	2	24	13	14	168	9436
4	1	3	16	13	21	112	8162
5	1	4	12	13	28	84	7616
6	1	6	8	13	42	56	7252
7	2	1	24	26	7	168	11452
8	2	2	12	26	14	84	7448
9	2	3	8	26	21	56	6356
10	2	4	6	26	28	42	5992
11	3	1	16	39	7	112	10850
12	3	2	8	39	14	56	7028
13	3	4	4	39	28	28	5936
14	4	1	12	52	7	84	10640
15	4	2	6	52	14	42	7000
16	4	3	4	52	21	28	6272
17	6	1	8	78	7	56	10612
18	6	2	4	78	14	28	7336
19	8	1	6	104	7	42	10780
20	8	2	3	104	14	21	7868
21	12	1	4	156	7	28	11312
22	12	2	2	156	14	14	9128
23	24	1	2	312	7	14	13300
24	48	1	1	624	7	7	17570

You can sort this spreadsheet on the Surface Area column to make picking the smallest value easier. Just select all of the cells with data and choose Sort from the Data menu. Be sure to choose the Surface Area column to sort by.

	A	B	C	D	E	F	G
1	Height (in Cans)	Length (in Cans)	Width (in Cans)	Height (in cm)	Length (in cm)	Height (in cm)	Surface Area (in square cm)
2	3	4	4	39	28	28	5936
3	2	4	6	26	28	42	5992
4	4	3	4	52	21	28	6272
5	2	3	8	26	21	56	6356
6	4	2	6	52	14	42	7000
7	3	2	8	39	14	56	7028
8	1	6	8	13	42	56	7252
9	6	2	4	78	14	28	7336
10	2	2	12	26	14	84	7448
11	1	4	12	13	28	84	7616
12	8	2	3	104	14	21	7868
13	1	3	16	13	21	112	8162
14	12	2	2	156	14	14	9128
15	1	2	24	13	14	168	9436
16	6	1	8	78	7	56	10612
17	4	1	12	52	7	84	10640
18	8	1	6	104	7	42	10780
19	3	1	16	39	7	112	10850
20	12	1	4	156	7	28	11312
21	2	1	24	26	7	168	11452
22	24	1	2	312	7	14	13300
23	1	1	48	13	7	336	13622
24	48	1	1	624	7	7	17570
25							

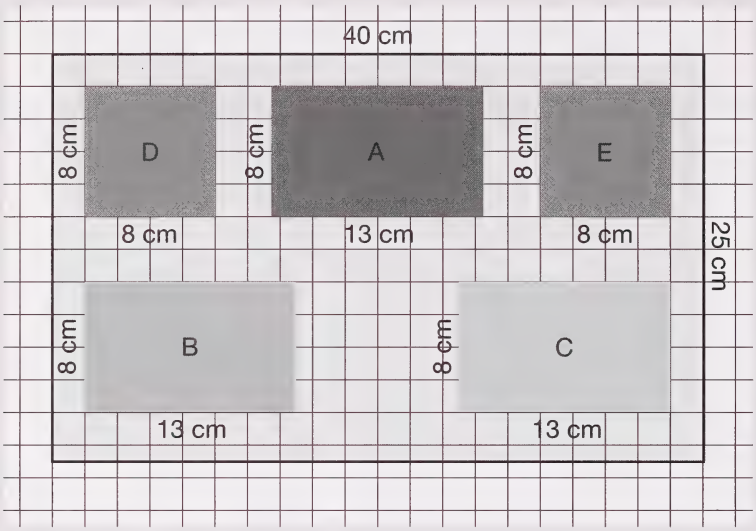
The smallest surface area for a box would be 5936 cm^2. You arrange the cans in 3 square layers of 16 cans.

Lesson 3: Design Problems

1. **Textbook, page 333, "Investigation 1: Designing a Notice Board"**

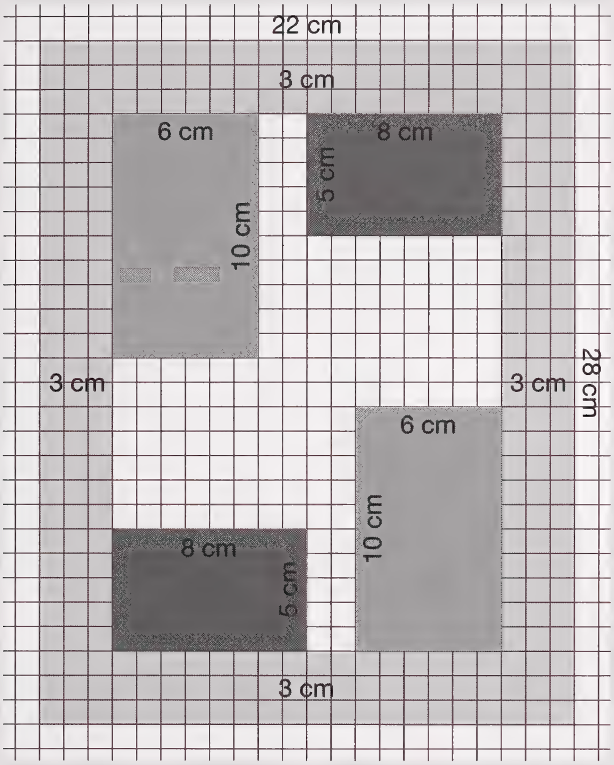
   Answers will vary. A sample answer follows. Notice these important features:

   - The width and height of the board are given on the diagram (40 cm and 25 cm).
   - The width and height of each coloured card are given on the diagram.
   - The cards are labelled with their letter names.

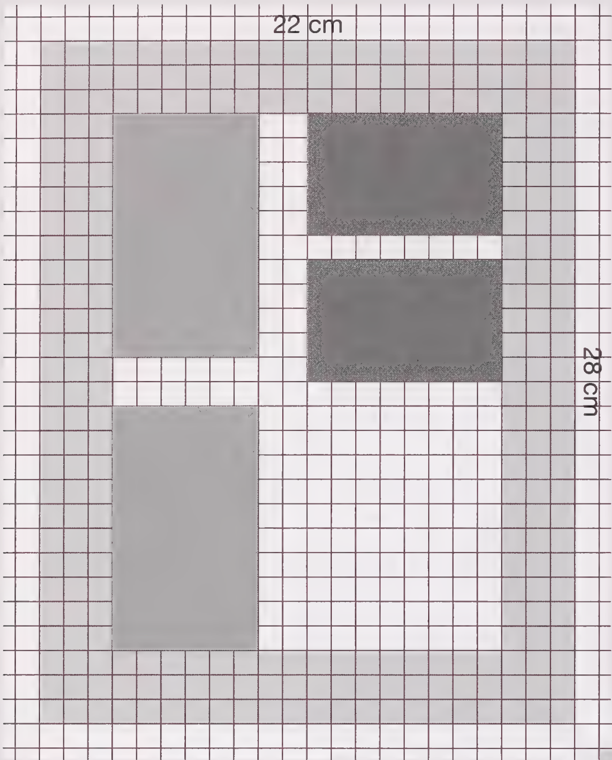


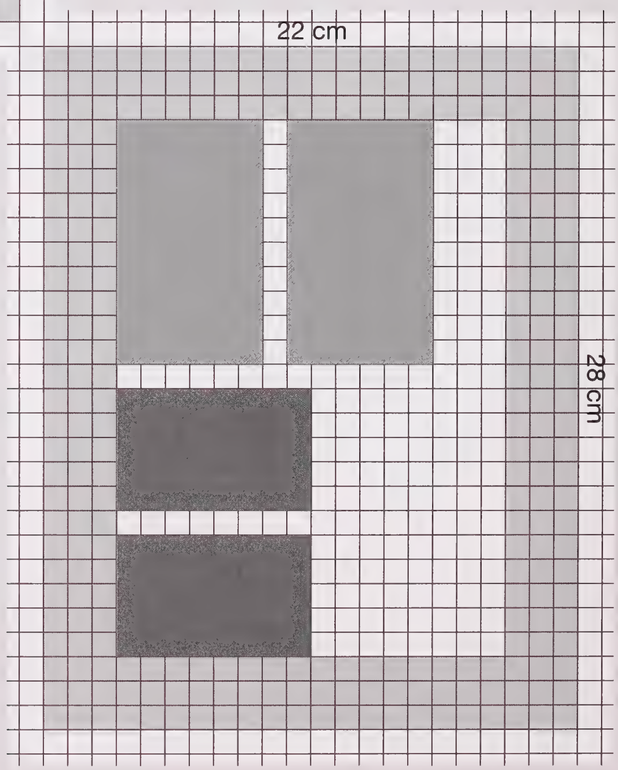
2. **Textbook, pages 334 to 336, "Put into Practice," questions 2, 3, 5, and 6**

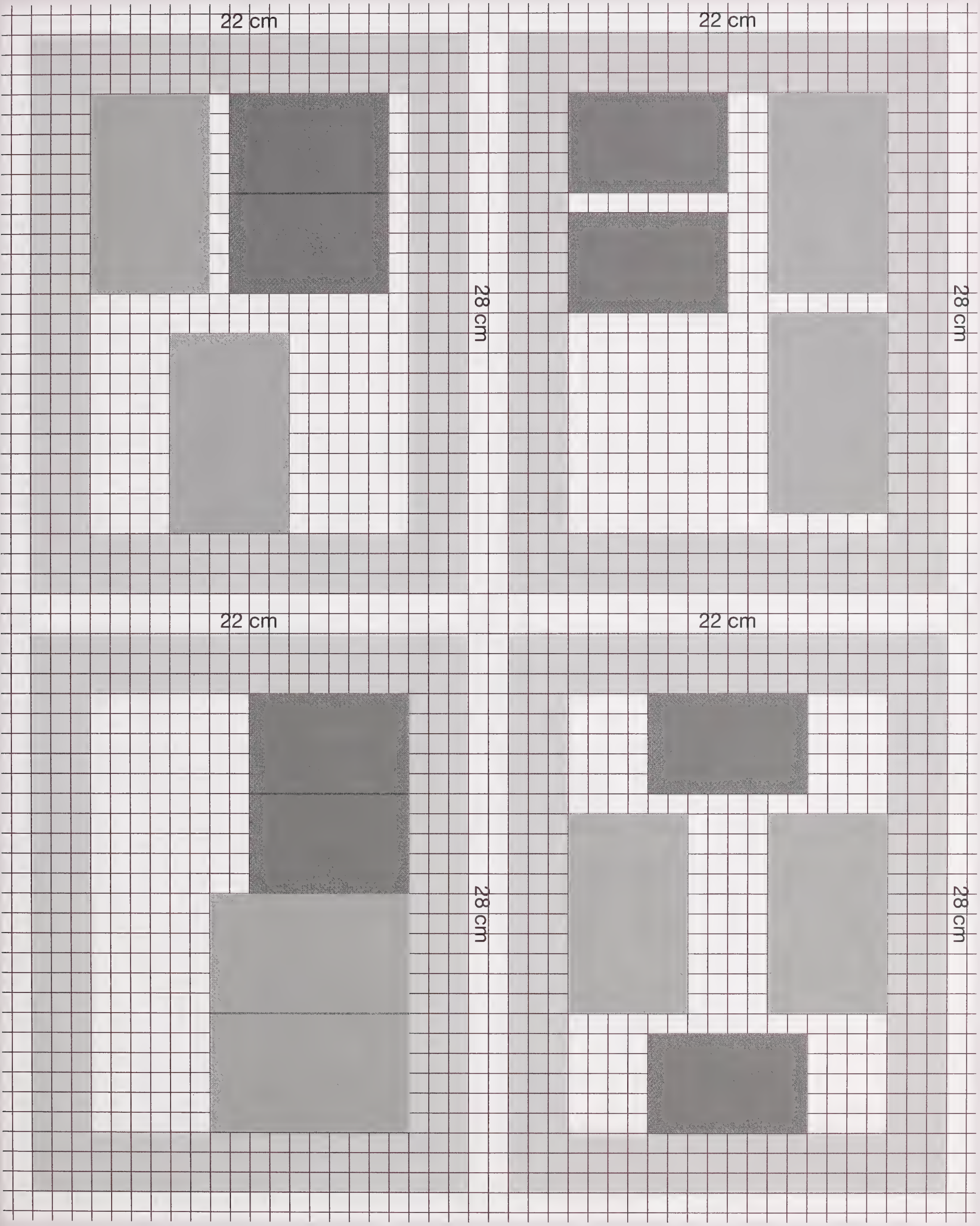
   2. **a.** Answers will vary. A sample answer follows.



There will be many ways to position the articles. Some other ways follow.






22 cm
28 cm
22 cm
28 cm
22 cm
28 cm
22 cm
28 cm

b. The usable area of the page is 352 cm^2.

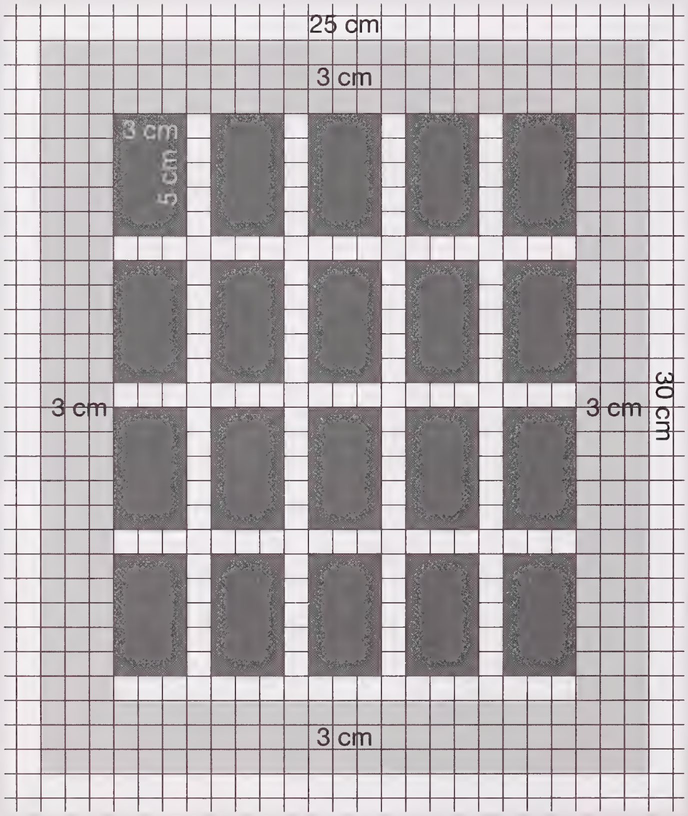
$$\begin{aligned}(22-6)\times(28-6) &= 16\times 22\\ &= 352\end{aligned}$$

The used area of the page is 200 cm^2.

$$\begin{aligned}2\times(6\times 10)+2\times(8\times 5) &= 120+80\\ &= 200\end{aligned}$$

The remaining usable area is 352 – 200 = 152 cm^2.

3. a. The following diagram shows that, at most, 20 pictures would fit on a page.



b. If one space was used for titles, there would be room for 19 pictures.

c. The following calculation shows that it would take 18 pages for all 328 pictures. There would be 17 full pages and another page with the remaining five pictures.

$$\begin{array}{r} 17 \\ 19\overline{)328} \\ \underline{19} \\ 138 \\ \underline{133} \\ 5 \end{array}$$

d. The remaining five pictures will be on the last page.

5. a. The perimeter consists of two straight sides and a semicircle. The length of the semicircle is half that of a circle with the same diameter.

$$\begin{aligned} C &= \pi d \\ &= \pi \times 4 \\ &\doteq 12.566\ 370\ 61 \end{aligned}$$

$$\begin{aligned} P &= s + s + \frac{C}{2} \\ &\doteq 7 + 7 + \frac{12.566\ 370\ 61}{2} \\ &\doteq 20.283\ 185\ 31 \end{aligned}$$

To the nearest metre, the perimeter of the pool is 20 m.

b. The area of the pool is the sum of the area of the semicircle and the isosceles triangle. Since the diameter of the semicircle is 4, the radius must be 2. You can use the value 6.7 given in the diagram for the height of the triangle. (Or you can use the Pythagorean Theorem to calculate it for yourself.)

$$\begin{aligned} A &= \frac{\pi r^2}{2} \\ &= \frac{\pi (2)^2}{2} \\ &\doteq 6.283\ 185\ 307 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2} bh \\ &= \frac{1}{2} \times 4 \times 6.7 \\ &= 13.4 \end{aligned}$$

$$\left(\begin{aligned} c^2 &= a^2 + b^2 \\ 7^2 &= 2^2 + b^2 \\ b &= \sqrt{49 - 4} \\ b &\doteq 6.708\ 203\ 932 \end{aligned} \right)$$

Since 6.283 185 307 + 13.4 = 19.683 185 31, the surface area of the pool is 20 m^2, rounded to the nearest square metre.

6. **a.** The large single disc of chocolate has a radius of 6 cm.

$$A = \pi r^2$$
$$= \pi (6)^2$$
$$\doteq 113.097\ 335\ 5$$

This gives a volume of 133.097 335 5 cm^3.

Each small chocolate disc has a radius of 3 cm.

$$A = \pi r^2$$
$$= \pi (3)^2$$
$$\doteq 28.274\ 333\ 88$$

The volume of four such discs is $4 \times 1 \times 28.274\ 333\ 88 = 113.097\ 335\ 5$ cm^3.

Both boxes contain the same amount of chocolate, 113 cm^3 to the nearest cubic centimetre.

b. Either box gives the same amount of chocolate. Your reason should deal with whether you want one large chocolate or four smaller ones.

3. **Textbook, page 337, "Investigation 2: Hub Cap Design," questions a to c**

a. Only wheels iii and iv have a line of symmetry.

iii. Try a diameter that passes through the middle of a spoke.

iv. Try a diameter that passes through the middle of a spoke or through the middle of the arc between spokes.

b. All four wheels have rotational symmetry.

i. Rotate the wheel 60° or multiples of 60°. (There are six points on this hubcap and $360 \div 6 = 60$.)

ii. Rotate the wheel 90° or multiples of 90°. (There are four points on this hubcap and $360 \div 4 = 90$.)

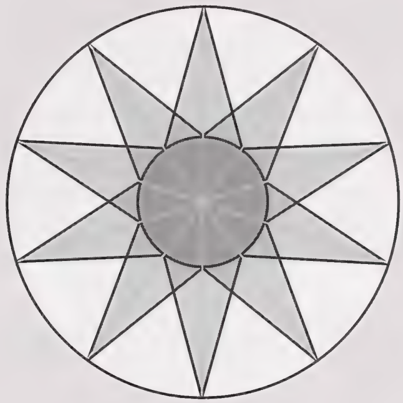
iii. Rotate the wheel 72° or multiples of 72°. (There are five arms on this hubcap and $360 \div 5 = 72$.)

iv. Rotate the wheel 45° or multiples of 45°. (There are eight points on this hubcap and $360 \div 8 = 45$.)

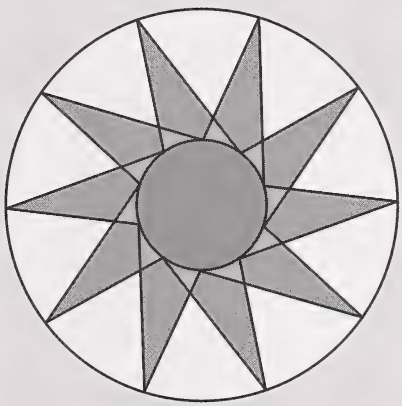
c. Wheels iii and iv have both a line of symmetry and rotational symmetry.

4. Textbook, page 339 "Put into Practice 2," questions 8 and 9

8. You could make a large number of designs. A sample ten-point design follows.



9. You could make a large number of different designs. A sample ten-point design follows.



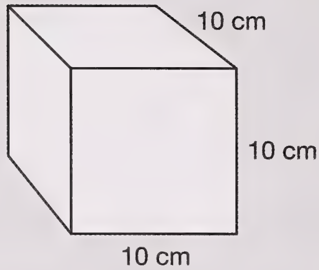
Review

Textbook, pages 340 to 343, "Review of Unit Six," questions 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, and 14

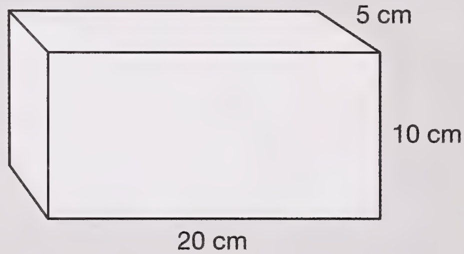
1. **a.** This is a rectangular prism. The base is a rectangle and all of the sides are rectangles.
   b. This is a rectangular pyramid. The base is a rectangle and all of the sides are triangles.
   c. This is a cone. The base is a circle and the sides taper to a point.
   d. This is an octagonal prism. The base is an octagon and the sides are rectangles.

2. Answers will vary. Sample answers are given.

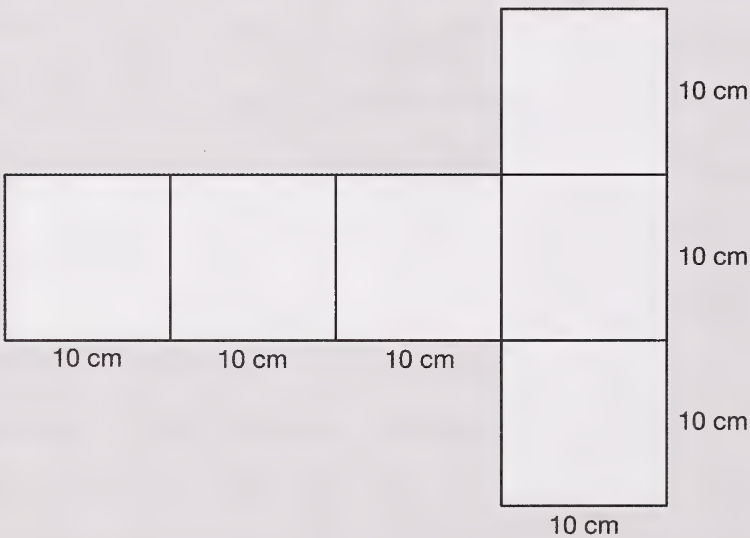
a. i. This box is 10 cm by 10 cm by 10 cm.



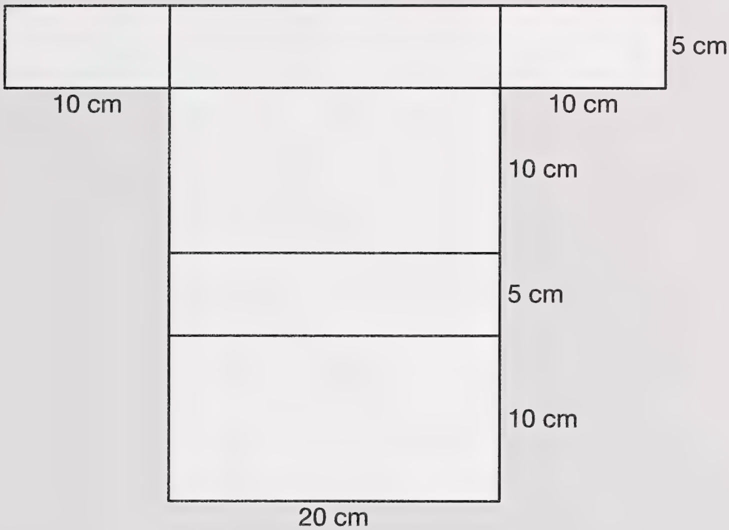
ii. This box is 20 cm by 5 cm by 10 cm.



b. i. This is a net for the cubic box.



ii. This is a net for the rectangular prism box.



c. **i.** The surface area of the cubic box is 600 cm^2.

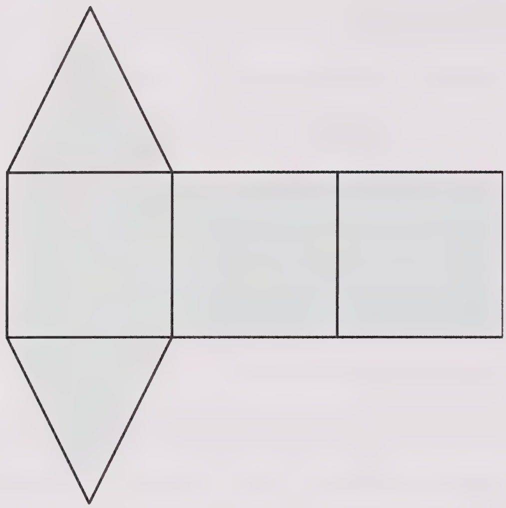
$$\begin{aligned} \text{surface area} &= 2\times[(\ell\times w)+(w\times h)+(h\times\ell)] \\ &= 2\times[(10\times10)+(10\times10)+(10\times10)]\ \text{cm}^2 \\ &= 2\times300\ \text{cm}^2 \\ &= 600\ \text{cm}^2 \end{aligned}$$

ii. The surface area of the rectangular box is 700 cm^2.

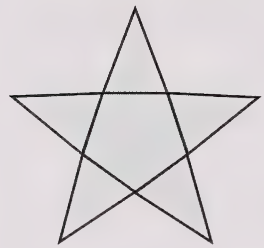
$$\begin{aligned} \text{surface area} &= 2\times[(\ell\times w)+(w\times h)+(h\times\ell)] \\ &= 2\times[(20\times10)+(10\times5)+(5\times20)]\text{cm}^2 \\ &= 2\times350\ \text{cm}^2 \\ &= 700\ \text{cm}^2 \end{aligned}$$

Box i has the smaller surface area.

3. **Triangular Prism**



Pentagonal Pyramid



4. a. This would be a cylinder.
   b. This would be an octagonal pyramid.
   c. This would be a triangular pyramid.
   d. This would be a hexagonal prism.

5. a. It would take 16 toothpicks to make a skeleton of the shape in question 4.b. Eight toothpicks would be needed to make the base. One toothpick would have to rise from each vertex of the base to the point of the pyramid. This would take eight more toothpicks, for a total of 16.

   It would take 18 toothpicks to make a skeleton of the shape in question 4.d. Six would be needed for the base and six more for the top. One would join a vertex on the base to the corresponding vertex on the top. This would take six more toothpicks, for a total of 18.

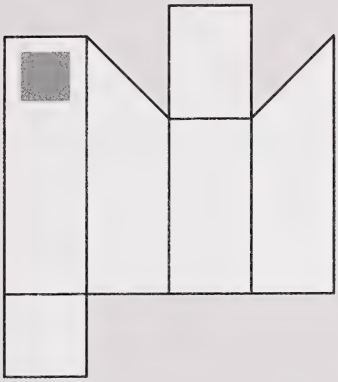
   b. The skeleton for shape 4.b. would have nine vertices, one at the top and eight around the base.

   The skeleton for shape 4.d. would have 12 vertices, one at each corner of the hexagons at the top and bottom.

   c. The skeleton for shape 4.b. would have 16 edges.

   The skeleton for shape 4.d. would have 18 edges.

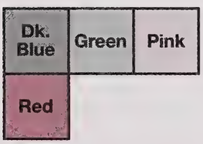
7. a.



b. **Front View**

Right View

8. a. **Top**



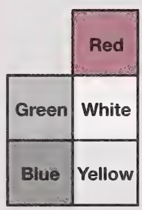
Front

Right

b. **Top**

Front

Right



10. a. **Volume**

$$V = \ell \times w \times h$$
$$= 24 \times 18 \times 9\ \text{cm}^3$$
$$= 3888\ \text{cm}^3$$

Surface Area

$$SA = 2 \times \left[(\ell \times w) + (w \times h) + (h \times \ell)\right]$$
$$= 2 \times \left[(24 \times 18) + (18 \times 9) + (9 \times 24)\right]\text{cm}^2$$
$$= 2 \times 810\ \text{cm}^2$$
$$= 1620\ \text{cm}^2$$

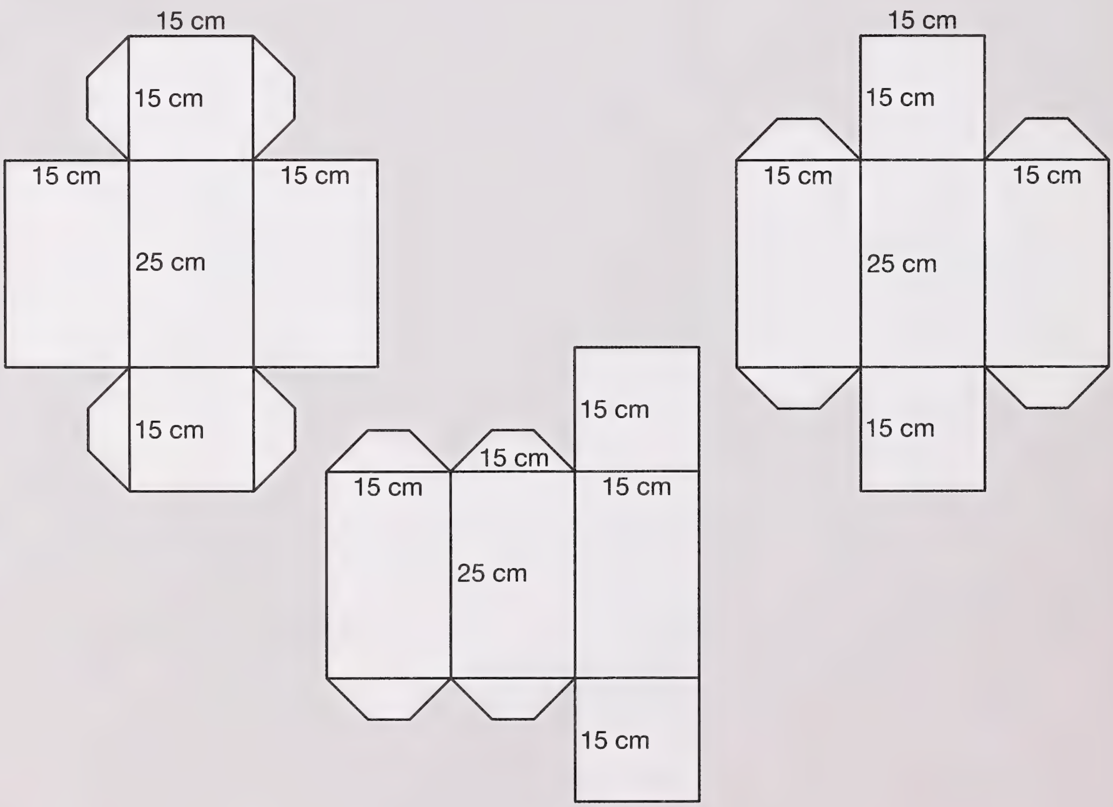
b. **Volume**

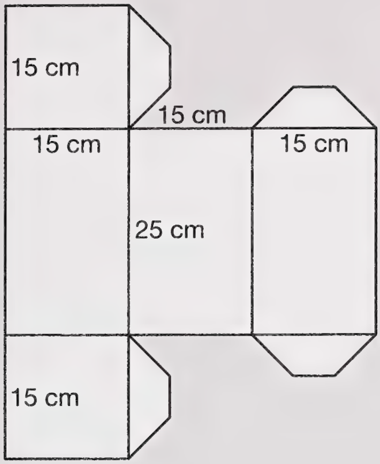
$$V = \pi r^2 h$$
$$= \pi \times 9^2 \times 34\ \text{cm}^3$$
$$\doteq 8651.946\ 168\ \text{cm}^3$$

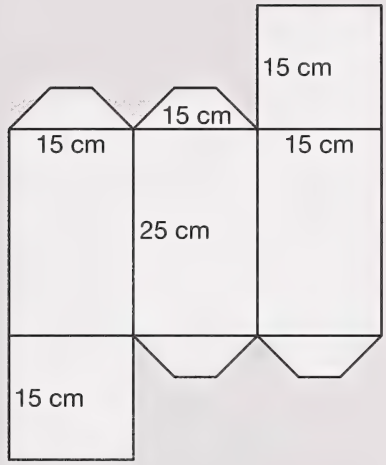
Surface Area

$$SA = 2\pi\left(r^2 + rh\right)$$
$$= 2\pi \times \left[9^2 + (9 \times 34)\right]\ \text{cm}^2$$
$$= 2\pi \times 387\ \text{cm}^2$$
$$\doteq 2431.592\ 714\ \text{cm}^2$$

12. Here are some nets for the CD storage box. The placement of the square ends and the flaps can vary considerably.







b. The amount of waste will depend on the size of the flaps. If you ignore the flaps, the waste can be calculated as follows.

Area of the Piece of Cardboard

$$\begin{aligned} A &= \ell \times w \\ &= 60 \text{ cm} \times 45 \text{ cm} \\ &= 2700 \text{ cm}^2 \end{aligned}$$

Amount Used for the Box

$$\begin{aligned} \text{SA} &= 3 \times (\ell \times w) + 2 \times (w \times h) \\ &= 3 \times (25 \times 15) \text{ cm}^2 + 2 \times (15 \times 15) \text{ cm}^2 \\ &= \left(3 \times 375 \text{ cm}^2\right) + \left(2 \times 225 \text{ cm}^2\right) \\ &= 1125 + 450 \text{ cm}^2 \\ &= 1575 \text{ cm}^2 \end{aligned}$$

The amount wasted is $2700 \text{ cm}^2 - 1575 \text{ cm}^2 = 1125 \text{ cm}^2$.

13. a. Volume of Rectangular Box

$$\begin{aligned} V &= \ell \times w \times h \\ &= 7 \times 20 \times 7 \text{ cm}^3 \\ &= 980 \text{ cm}^3 \end{aligned}$$

Volume of Cylindrical Box

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 3.5^2 \times 20 \text{ cm}^3 \\ &\doteq 769.690\,200\,1 \text{ cm}^3 \end{aligned}$$

To the nearest cubic centimetre, the rectangular box has a volume of 980 cm^3 and the cylindrical box has a volume of 770 cm^3.

b. **Surface Area of Rectangular Box**

$$\begin{aligned}SA &= 2\times[(\ell\times w)+(w\times h)+(h\times \ell)]\\ &= 2\times[(20\times 7)+(7\times 7)+(7\times 20)]\text{ cm}^2\\ &= 2\times 329\text{ cm}^2\\ &= 658\text{ cm}^2\end{aligned}$$

Surface Area of Cylindrical Box

$$\begin{aligned}SA &= 2\pi(r^2+rh)\\ &= 2\pi\times(3.5^2+3.5\times 20)\text{ cm}^2\\ &= 2\pi\times 82.25\text{ cm}^2\\ &\doteq 516.791\,991\,5\text{ cm}^2\end{aligned}$$

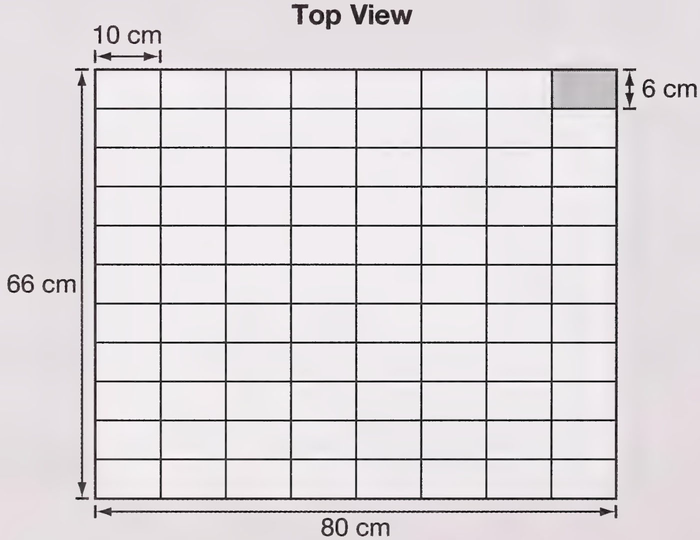
To the nearest square centimetre, the rectangular box has a surface area of 658 cm^2 and the cylindrical box has a surface area of 517 cm^2.

c. There would be $980\text{ cm}^3 - (3\times 150)\text{cm}^3 = 530\text{ cm}^3$ of wasted space in the rectangular box. There would be $770\text{ cm}^3 - (3\times 150)\text{cm}^3 = 320\text{ cm}^3$ of wasted space in the cylindrical box.

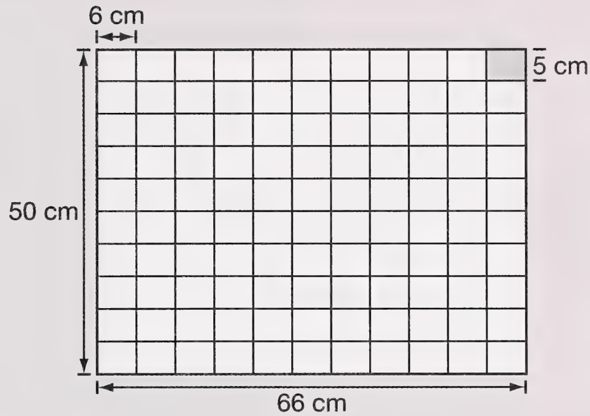
d. The rectangular box will cost 658 cm^2 × \$0.004/cm^2 = \$2.632 or, to the nearest cent, \$2.63. The cylindrical box will cost 517 cm^2 × \$0.004/cm^2 = \$2.068 or, to the nearest cent, \$2.07.

14. You can pack 10 layers of small boxes into the shipping box. Each box would be sitting on a 10 cm-by-6 cm side and rising upward 5 cm. If the small boxes are positioned correctly, you can place 8 rows of 11 boxes in each layer. This is shown in the following views.

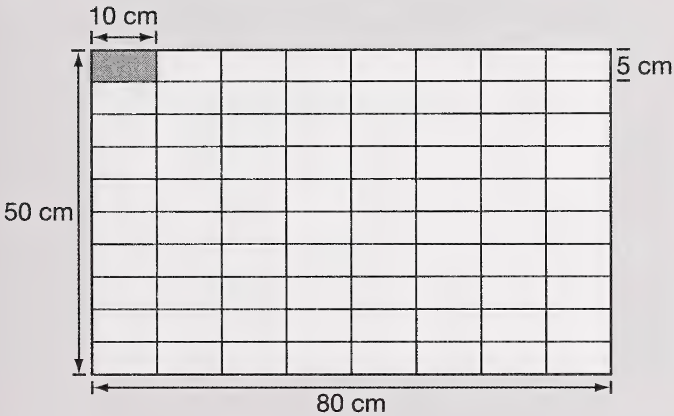
Top View



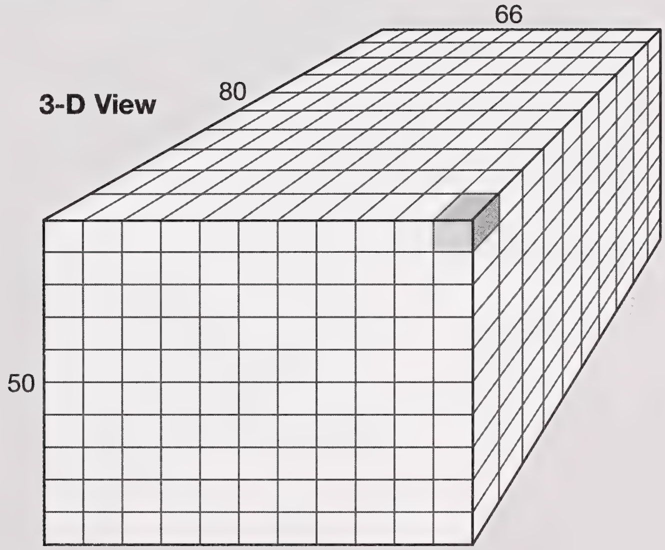
Front View



Right View



3-D View



This gives a total of $10 \times 8 \times 11 = 880$ small boxes of soap in the large shipping box.

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18 Brand X Pictures/Getty Images
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Module 6 Spreadsheet Templates and Multimedia

The following multimedia segments and spreadsheet template for Module 6 appear on the multimedia CD:

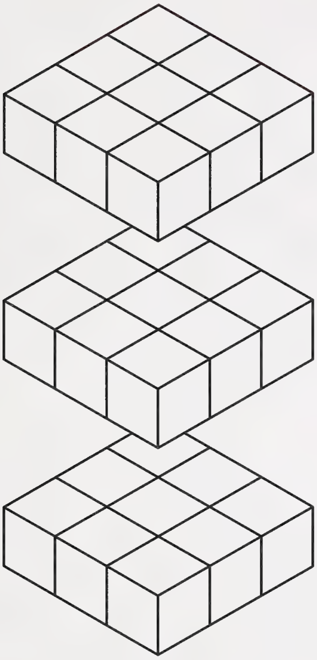
- "Converting from Nets to Solids and Solids to Nets"
- "Module 6: Section 2: Lesson 1 and Related Diagrams"
- "Viewing 3-D Objects"
- Mod_6_1.xlt

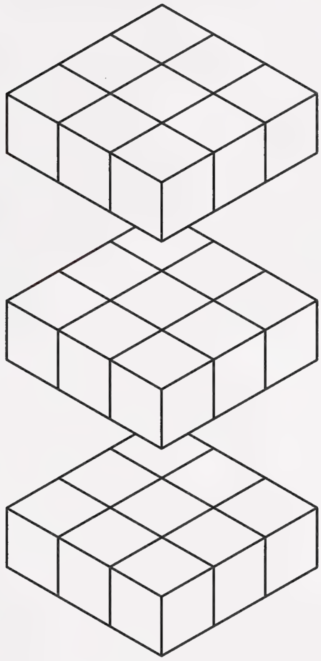
Learning Aids

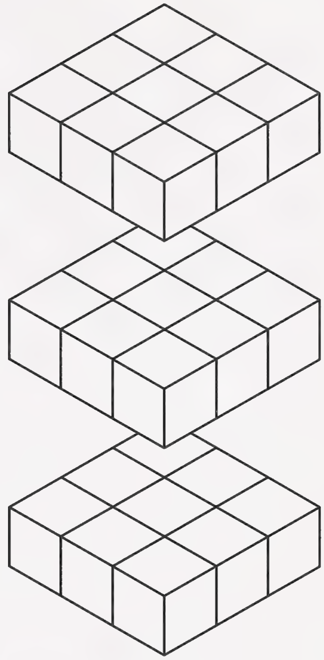
Isometric Grid Paper

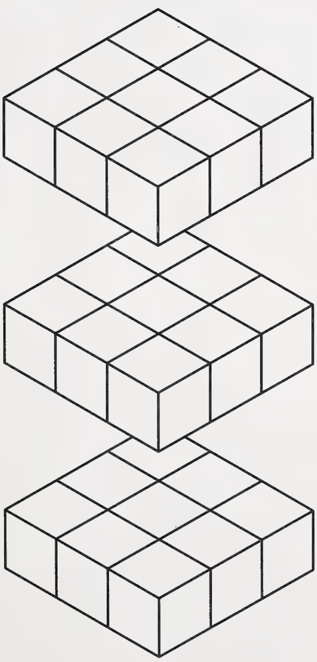
Cartesian Grid Paper

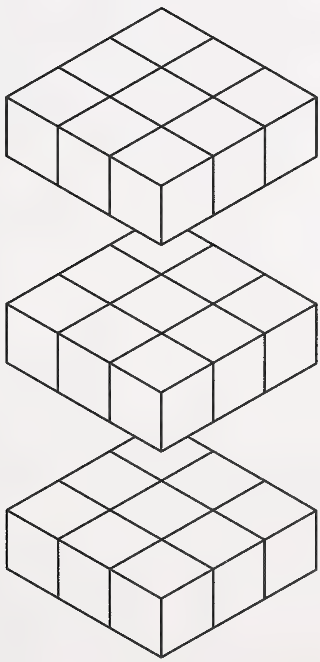
Stack Diagrams

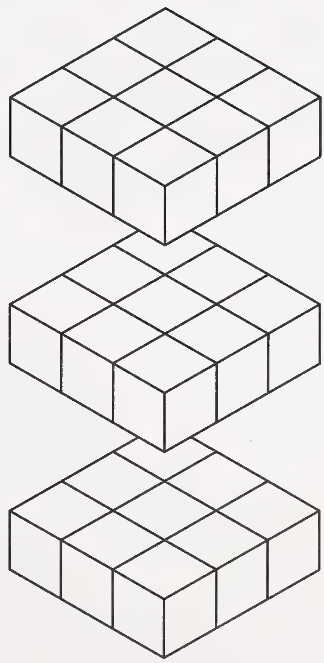




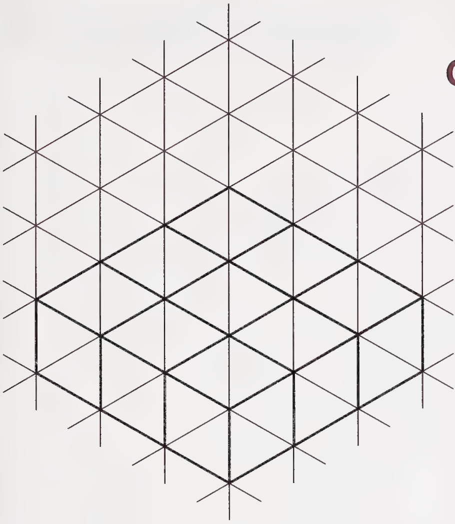


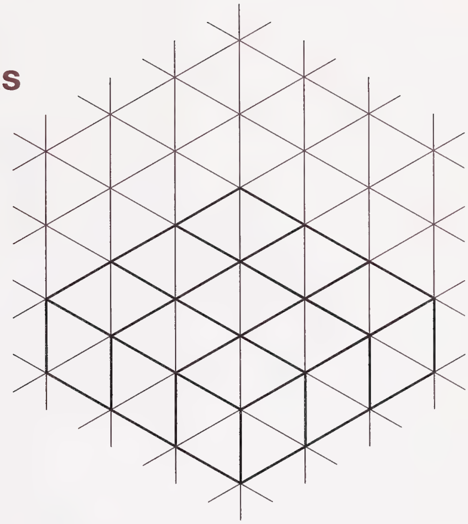


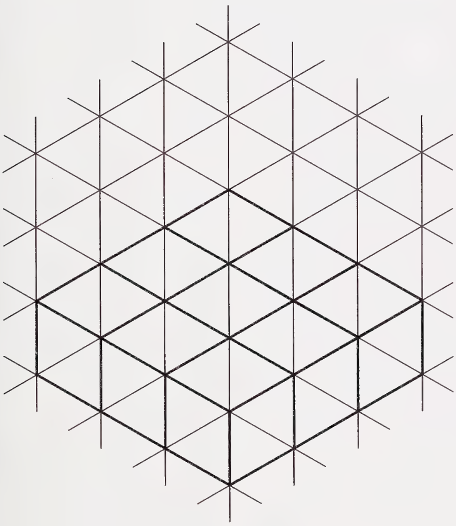


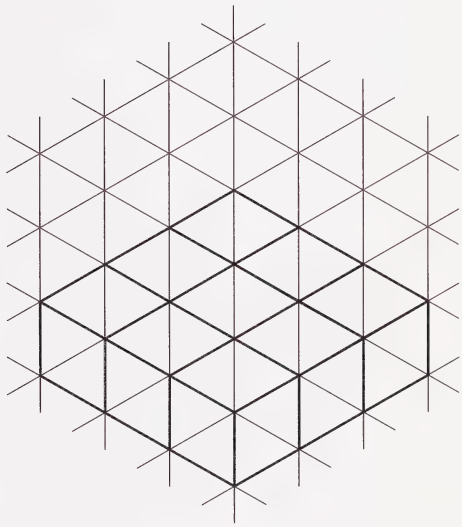


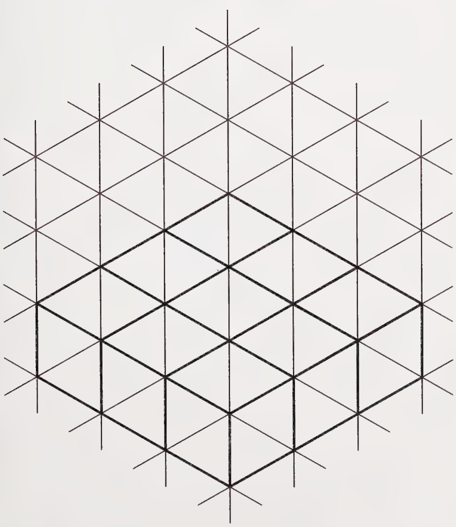
Grids for 3-D Cube Diagrams

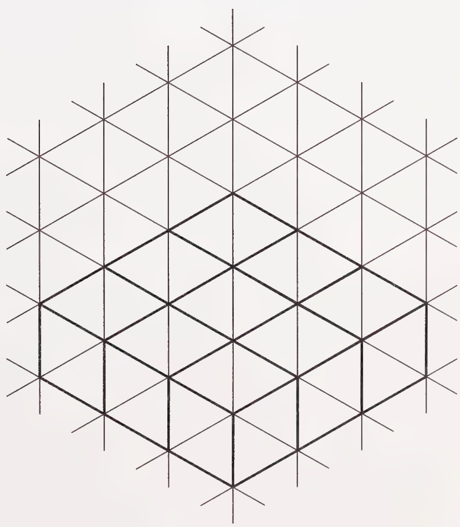




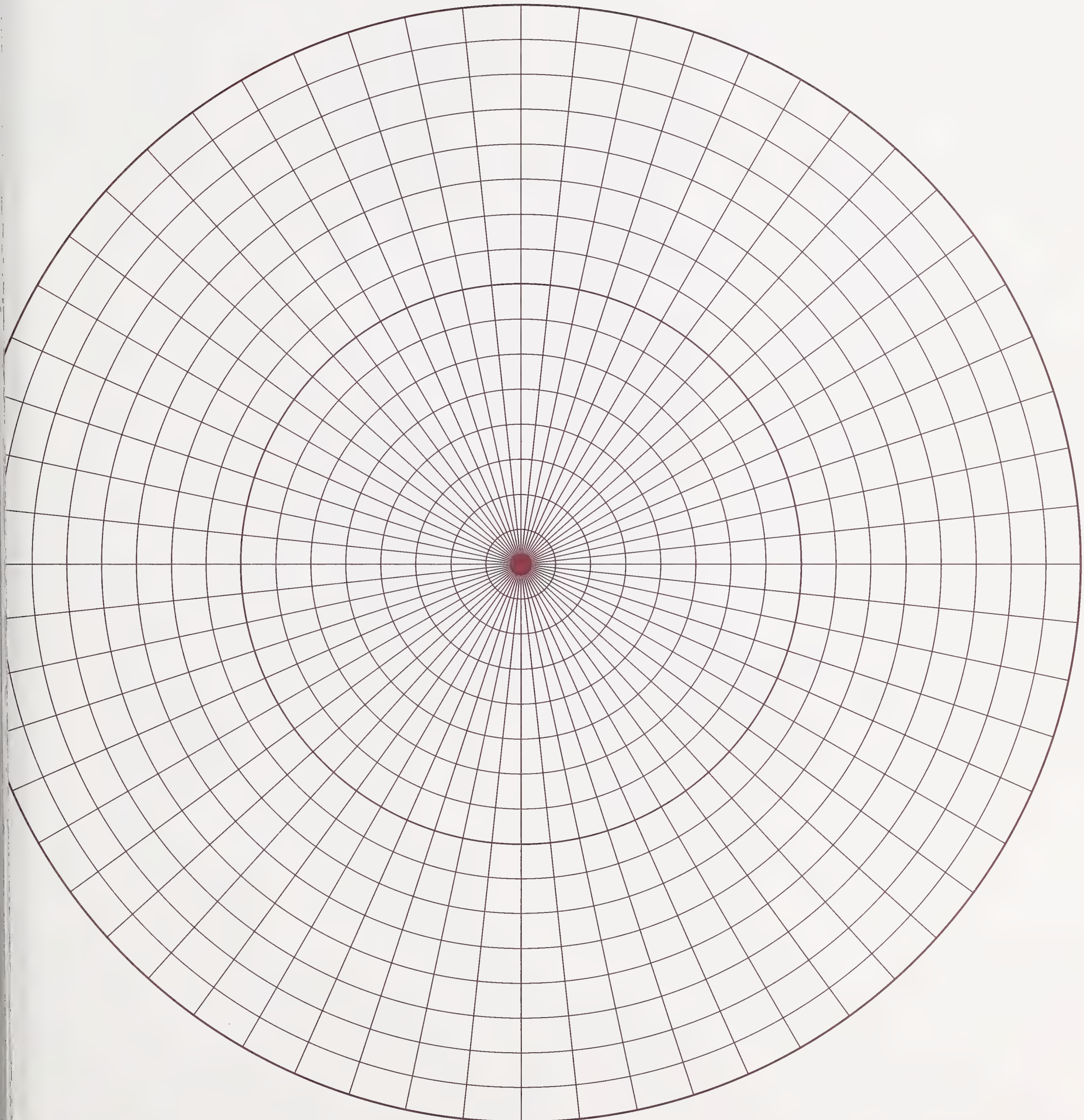




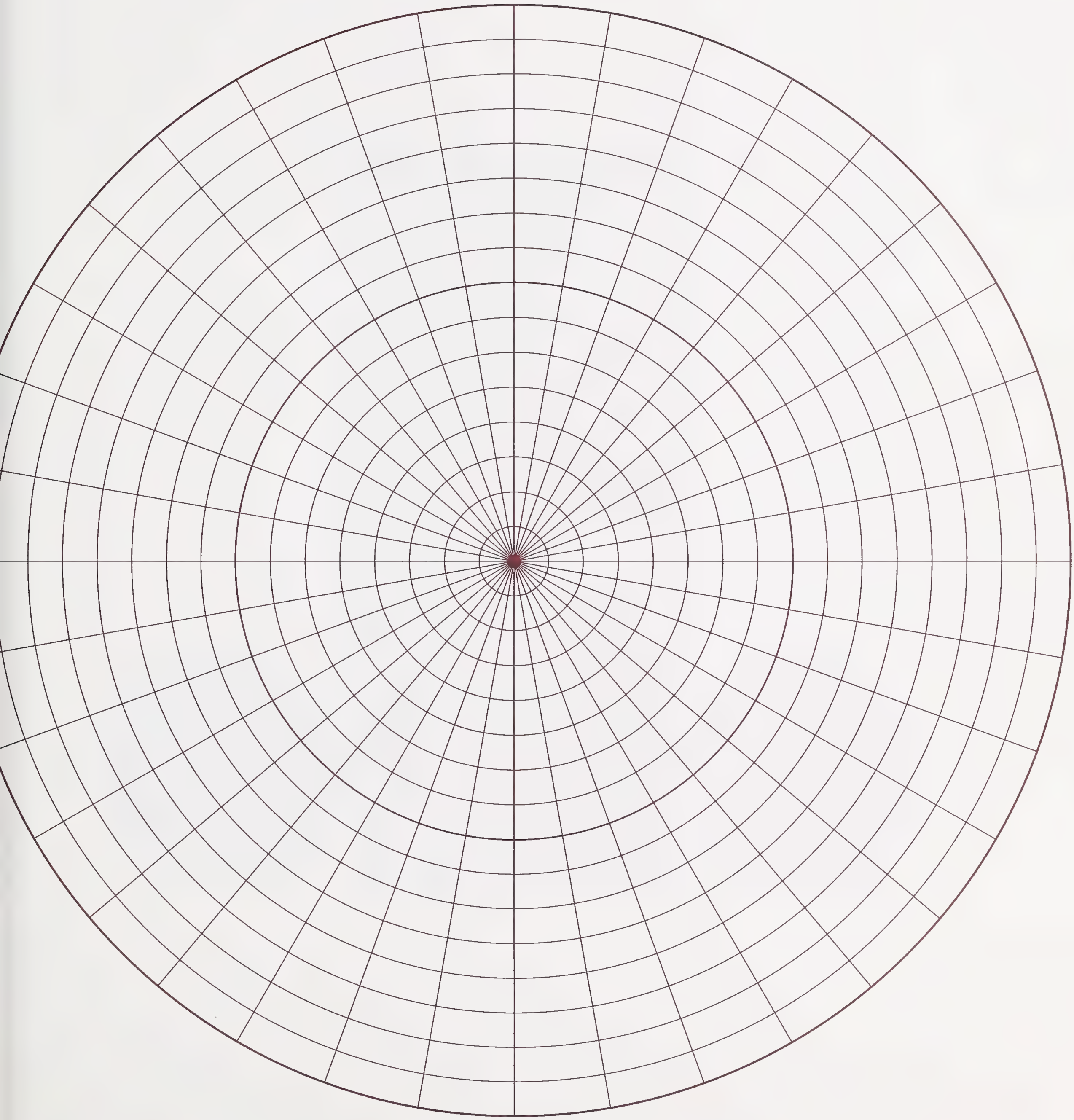




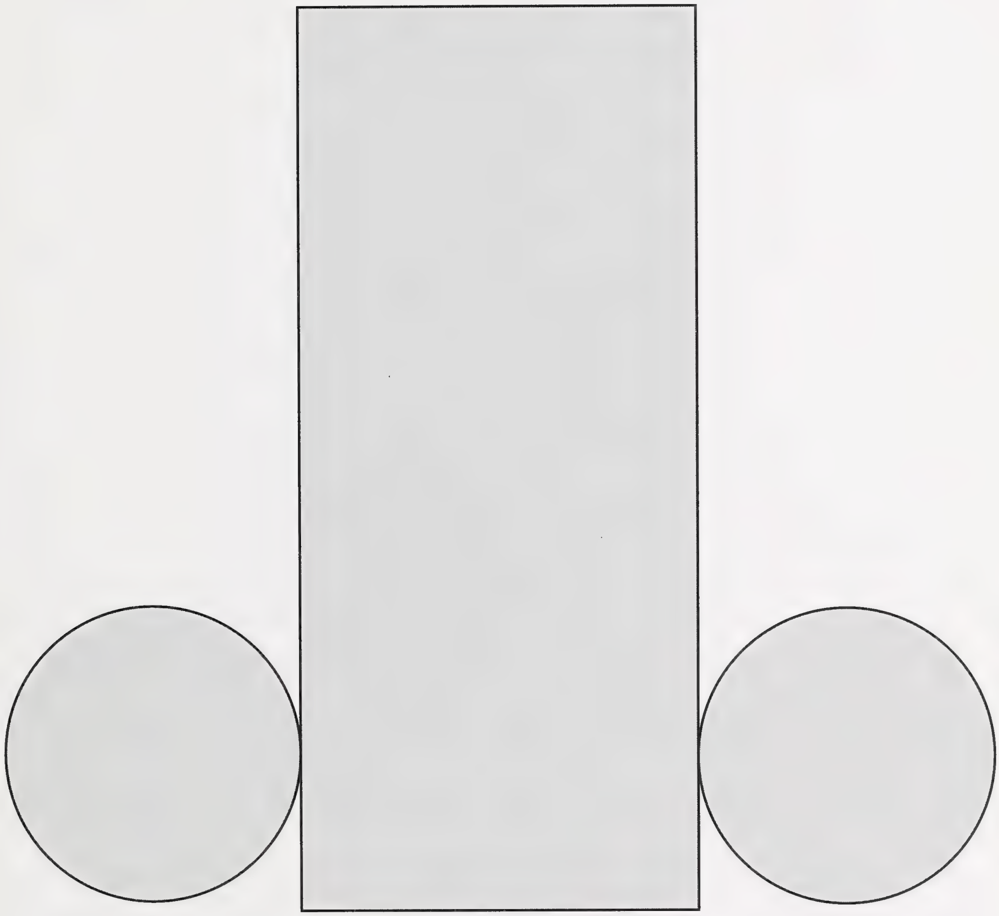
6° Circular Grid Paper



10° Circular Grid Paper



Nets



Nets